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Slant Riemannian submersions from Sasakian manifolds

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Abstract. We introduce and characterize slant Riemannian submersions from Sasakian manifolds onto Riemannian manifolds. We survey main results of slant Riemannian submersions defined on Sasakian manifolds. We give a sufficient condition for a slant Riemannian submersion from Sasakian manifolds onto Riemannian manifolds to be harmonic. We also give an example of such slant submersions. Moreover, we find a sharp inequality between the scalar curvature and norm squared mean curvature of fibres.

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1. INTRODUCTION

Let F be a C^∞-submersion from a Riemannian manifold (M, g_M) onto a Riemannian manifold (N, g_N). Then according to the conditions on the map $F : (M, g_M) \to (N, g_N)$, F can be any one of the following types: semi-Riemannian submersion and Lorentzian submersion [11], Riemannian submersion [22,12], slant submersion [9,27], almost Hermitian submersion [29], contact-complex submersion [13], quaternionic submersion [14], almost h-slant submersion and h-slant submersion [24], semi-invariant submersion [28], h-semi-invariant submersion [25], etc.

As we know, Riemannian submersions are related to physics and have their applications in the Yang–Mills theory [6,30], Kaluza–Klein theory [7,15], supergravity and superstring theories [16,21]. In [26], Şahin introduced anti-invariant Riemannian submersions from almost Hermitian manifolds onto Riemannian manifolds. He gave a generalization of Hermitian

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submersions and anti-invariant submersions by defining and studying slant submersions from almost Hermitian manifolds onto Riemannian manifolds [27].

The present work is another step in this direction, more precisely from the point of view of slant Riemannian submersions from Sasakian manifolds. We also want to carry anti-invariant submanifolds of Sasakian manifolds to anti-invariant Riemannian submersion theory and to prove dual results for submersions. For instance, a slant submanifold of a K-contact manifold is an anti invariant submanifold if and only if $\nabla Q = 0$ (see Proposition 4.1 of [8]). We get a result similar to Proposition 4. Although slant submanifolds of contact metric manifolds were studied by several different authors and are considered a well-established topic in contact Riemannian geometry, only little about slant submersions are known. So, we study slant Riemannian submersions from almost contact metric manifolds onto Riemannian manifolds. Recently, the authors in [17,20] and [18] studied anti-invariant Riemannian submersions from almost contact manifolds independently of each other.

The paper is organized as follows: In Section 2, we present the basic information about Riemannian submersions needed throughout this paper. In Section 3, we mention about Sasakian manifolds. In Section 4, we give the definition of slant Riemannian submersions and introduce slant Riemannian submersions from Sasakian manifolds onto Riemannian manifolds. We survey main results on slant submersions defined on Sasakian manifolds. We give a sufficient condition for a slant Riemannian submersion from Sasakian manifolds onto Riemannian manifolds to be harmonic. Moreover, we investigate the geometry of leaves of $(\ker F_*)$ and $(\ker F_*)^\perp$. We give an example of slant submersions such that the characteristic vector field ξ is vertical. Moreover, we find a sharp inequality between the scalar curvature and squared mean curvature of fibres.

2. Riemannian submersions

In this section we recall several notions and results which will be needed throughout the paper.

Let (M, g_M) be an m-dimensional Riemannian manifold and let (N, g_N) be an n-dimensional Riemannian manifold. A Riemannian submersion is a smooth map $F : M \to N$ which is onto and satisfies the following axioms:

$S1$. F has maximal rank.

$S2$. The differential F_* preserves the lengths of horizontal vectors.

The fundamental tensors of a submersion were defined by O'Neill [22], [23]. They are $(1,2)$-tensors on M, given by the following formulas:

$$\mathcal{T}(E,F) = \mathcal{T}_E F = \mathcal{H}\nabla_{\mathcal{V}E}\mathcal{V}F + \mathcal{V}\nabla_{\mathcal{V}E}\mathcal{H}F, \tag{2.1}$$

$$\mathcal{A}(E,F) = \mathcal{A}_E F = \mathcal{V}\nabla_{\mathcal{H}E}\mathcal{H}F + \mathcal{H}\nabla_{\mathcal{H}E}\mathcal{V}F, \tag{2.2}$$

for any vector fields E and F on M. Here ∇ denotes the Levi-Civita connection of (M, g_M). These tensors are called integrability tensors for the Riemannian submersions. Here we denote the projection morphism on the distributions $\ker F_*$ and $(\ker F_*)^\perp$ by $\mathcal{V}$ and $\mathcal{H}$, respectively. The following lemmas are well known [22,23]:

Lemma 1. *For any U, W vertical and X, Y horizontal vector fields, the tensor fields $\mathcal{T}$ and $\mathcal{A}$ satisfy*

$$\text{(i)}\, \mathcal{T}_U W = \mathcal{T}_W U, \tag{2.3}$$