ARAB JOURNAL OF
MATHEMATICAL SCIENCES

Arab J Math Sci 22 (2016) 265–274

Values shared by meromorphic functions and their derivatives

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Received 31 December 2014; received in revised form 18 December 2015; accepted 16 January 2016
Available online 3 February 2016

Abstract. In this paper we deal with the problem of uniqueness of meromorphic functions as well as their power which share a small function with their derivatives and obtain some results which improve and generalize the recent results due to Zhang and Yang (2009) and Sheng and Zongsheng (2012).

2010 Mathematics Subject Classification: 30D35

Keywords: Meromorphic function; Derivative; Small function

1. INTRODUCTION DEFINITIONS AND RESULTS

In this paper, by a meromorphic function we will always mean a meromorphic function in the complex plane $\mathbb{C}$. We adopt the standard notations of Nevanlinna theory of meromorphic functions as explained in [4]. It will be convenient to let E denote any set of positive real numbers of finite linear measure, not necessarily same at each occurrence. For a non-constant meromorphic function h, we denote by $T(r,h)$ Nevanlinna characteristic function of h and by $S(r,h)$ any quantity satisfying $S(r,h) = o\{T(r,h)\}$, as $r \longrightarrow \infty$ and $r \notin E$.

Let k be a positive integer and $a \in \mathbb{C} \cup \{\infty\}$. We use $N_{k)}(r,a;f)$ to denote counting function of a-points of f with multiplicity $\leq k$, $N_{(k+1}(r,a;f)$ to denote counting function of a-points of f with multiplicity $>k$. Similarly $\overline{N}_{k)}(r,a;f)$ and $\overline{N}_{(k+1}(r,a;f)$ are their reduced functions respectively.

Let f and g be two non-constant meromorphic functions and let a be a complex number. We say that f and g share a CM, provided that $f-a$ and $g-a$ have the same zeros with the same multiplicities. Similarly, we say that f and g share a IM, provided that $f-a$and $g-a$

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Peer review under responsibility of King Saud University.

ELSEVIER Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.ajmsc.2016.01.001

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have the same zeros ignoring multiplicities. In addition, we say that f and g share ∞ CM, if $1/f$ and $1/g$ share 0 CM, and we say that f and g share ∞ IM, if $1/f$ and $1/g$ share 0 IM.

A meromorphic function a is said to be a small function of f provided that $T(r,a) = S(r,f)$, that is $T(r,a) = o(T(r,f))$ as $r \longrightarrow \infty$, $r \notin E$.

During the last four decades uniqueness theory of entire and meromorphic functions has become a prominent branch of value distribution theory (see [12]).

Rubel–Yang [6] proposed to investigate uniqueness of an entire function f under the assumption that f and its derivative f' share two complex values. Subsequently, related to one or two value sharing similar considerations have been made with respect to higher derivatives and more general (linear) differential expressions by Brück [1], Gundersen [2], Mues–Steinmetz [5], Yang [8].

In this direction an interesting problem still open is the following conjecture proposed by Brück [1]:

Conjecture 1.1. *Let f be a non-constant entire function. Suppose*

$$\rho_1(f) := \limsup_{r\to\infty} \frac{\log\log T(r,f)}{\log r}$$

is not a positive integer or infinite. If f and f' share one finite value a CM, then

$$\frac{f' - a}{f - a} = c$$

for some non-zero constant c.

The case that $a = 0$ and that $N(r,0;f') = S(r,f)$ had been proved by Brück [1] while the case that f is of finite order had been proved by Gundersen–Yang [3]. However, the corresponding conjecture for meromorphic functions fails in general (see [3]).

To the knowledge of the author perhaps Yang–Zhang [10] (see also [13]) were the first to consider uniqueness of a power of a meromorphic (entire) function $F = f^n$ and its derivative F' when they share a certain value as this type of consideration gives the most specific form of the function.

As a result during the last decade, growing interest has been devoted to this setting of meromorphic functions. Improving all the results obtained in [10], Zhang [13] proved the following theorem.

Theorem A ([13])**.** *Let f be a non-constant meromorphic function, n, k be positive integers and $a(z)(\not\equiv 0,\infty)$ be a meromorphic small function of f. Suppose $f^n - a$ and $(f^n)^{(k)} - a$ share the value 0 CM and*

$$(n-k-1)(n-k-4) > 3k+6, \tag{1.1}$$

then $f^n \equiv (f^n)^{(k)}$, and f assumes the form

$$f(z) = ce^{\frac{\lambda}{n}z},$$

where c is a nonzero constant and $\lambda^k = 1$.

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