Arab Journal of Mathematical Sciences

Arab J Math Sci 21 (2015) 170–178

ϕ-semisymmetric generalized Sasakian space-forms

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Received 12 July 2013; received in revised form 18 November 2014; accepted 14 January 2015
Available online 12 February 2015

Abstract. The object of the present paper is to study ϕ-Weyl semisymmetric and ϕ-projectively semisymmetric generalized Sasakian space-forms. Finally, illustrative examples are given.

Keywords: Generalized Sasakian space-form; ϕ-Weyl semisymmetric manifold; ϕ-projectively semisymmetric manifold; Conformally flat; Projectively flat

2010 Mathematics Subject Classification: 53C15; 53C25

1. Introduction

The nature of a Riemannian manifold mostly depends on the curvature tensor R of the manifold. It is well known that the sectional curvatures of a manifold determine its curvature tensor completely. A Riemannian manifold with constant sectional curvature c is known as a real space-form and its curvature tensor is given by

$$R(X,Y)Z = c\{g(Y,Z)X - g(X,Z)Y\}.$$

A Sasakian manifold with constant ϕ-sectional curvature is a Sasakian space-form and it has a specific form of its curvature tensor. Similar notion also holds for Kenmotsu and cosymplectic space-forms. In order to generalize such space-forms in a common frame Alegre, Blair and Carriazo [1] introduced and studied generalized Sasakian space-forms. These space-forms are defined as follows:

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Peer review under responsibility of King Saud University.

ELSEVIER Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.ajmsc.2015.01.002

Given an almost contact metric manifold $M(\phi, \xi, \eta, g)$, we say that M is a generalized Sasakian space-form if there exist three functions f_1, f_2, f_3 on M such that the curvature tensor R is given by

$$\begin{aligned} R(X,Y)Z = {} & f_1\{g(Y,Z)X - g(X,Z)Y\} \\ & + f_2\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\} \\ & + f_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\ & + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\}. \end{aligned} \tag{1.1}$$

for any vector fields X, Y, Z on M. In such a case we denote the manifold as $M(f_1, f_2, f_3)$. In [1] the authors cited several examples of generalized Sasakian space-forms. If $f_1 = \frac{c+3}{4}$, $f_2 = \frac{c-1}{4}$ and $f_3 = \frac{c-1}{4}$, then a generalized Sasakian space-form with Sasakian structure becomes a Sasakian space-form. In [12], Kim studied conformally flat generalized Sasakian space-forms and locally symmetric generalized Sasakian space-forms. He proves some geometric properties of generalized Sasakian space-form which depends on the nature of the functions f_1, f_2 and f_3. A large number of geometers have studied generalized Sasakian space-forms in the papers [2,3,5,4,8]. In [9] De and Sarkar study locally ϕ-symmetric generalized Sasakian space-forms and generalized Sasakian space-forms with η-recurrent Ricci tensor. Also De and Sarkar [10] study projectively flat, projectively semisymmetric generalized Sasakian space-forms. Again in [16] Yildiz and De study ϕ-Weyl semisymmetric and ϕ-projectively semisymmetric non-Sasakian (k, μ)-contact metric manifolds. Motivated by these studies in this paper we study ϕ-Weyl semisymmetric and ϕ-projectively semisymmetric generalized Sasakian space-forms. The present paper is organized as follows:

After preliminaries in Section 3, we consider ϕ-Weyl semisymmetric generalized Sasakian space-forms and obtain necessary and sufficient conditions for a generalized Sasakian space-form to be ϕ-Weyl semisymmetric. Section 4 deals with ϕ-projectively semisymmetric generalized Sasakian space-forms. Finally, illustrative examples are given.

2. Preliminaries

In an almost contact metric manifold we have [6,7]

$$\phi^2(X) = -X + \eta(X)\xi, \quad \phi\xi = 0. \tag{2.1}$$

$$\eta(\xi) = 1, \qquad g(X,\xi) = \eta(X), \qquad \eta(\phi X) = 0. \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y). \tag{2.3}$$

$$g(\phi X, Y) = -g(X, \phi Y), \qquad g(\phi X, X) = 0. \tag{2.4}$$

$$g(\phi X, \xi) = 0. \tag{2.5}$$

Again we know that [1] in a $(2n+1)$-dimensional generalized Sasakian space-form:

$$\begin{aligned} R(X,Y)Z = {} & f_1\{g(Y,Z)X - g(X,Z)Y\} \\ & + f_2\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\} \\ & + f_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\ & + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\}. \end{aligned} \tag{2.6}$$

$$S(X,Y) = (2nf_1 + 3f_2 - f_3)g(X,Y) - (3f_2 + (2n-1)f_3)\eta(X)\eta(Y). \tag{2.7}$$

$$QX = (2nf_1 + 3f_2 - f_3)X - (3f_2 + (2n-1)f_3)\eta(X)\xi. \tag{2.8}$$