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Original article

Statistical convergence of order α in probability

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Abstract. In this paper the ideas of different types of convergence of a sequence of random variables in probability, namely, statistical convergence of order α in probability, strong p-Cesàro summability of order α in probability, lacunary statistical convergence or S_θ-convergence of order α in probability, and N_θ-convergence of order α in probability have been introduced and their certain basic properties have been studied.

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1. Introduction and background

The idea of convergence of a real sequence has been extended to statistical convergence by Fast [14] and Steinhaus [31] as follows: If $\mathbb{N}$ denotes the set of natural numbers and $K \subset \mathbb{N}$, then $K(m,n)$ denotes the cardinality of the set $K \cap [m,n]$. The upper and lower natural density of the subset K is defined by

$$\bar{d}(K) = \lim_{n\to\infty} \sup \frac{K(1,n)}{n} \quad \text{and} \quad \underline{d}(K) = \lim_{n\to\infty} \inf \frac{K(1,n)}{n}.$$

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If $\bar{d}(K) = \underline{d}(K)$, then we say that the natural density of K exists, and it is denoted simply by

$$d(K) = \lim_{n\to\infty} \frac{K(1,n)}{n}.$$

A sequence $\{x_n\}_{n\in\mathbb{N}}$ of real numbers is said to be statistically convergent to a real number x if for each $\varepsilon > 0$, the set $K = \{n \in \mathbb{N} : |x_n - x| \geq \varepsilon\}$ has natural density zero and we write $x_n \xrightarrow{S} x$. Statistical convergence has turned out to be one of the most active areas of research in summability theory after the work of Fridy [16] and Šalát [28]. Over the years a lot of work have been done to generalize this notion of statistical convergence and to introduce new summability methods related to it. Some of the most important concepts introduced are : lacunary statistical convergence by Fridy & Orhan [18] (for more results on this convergence see the paper of Li [24]), $\mathcal{I}$-convergence by Kostyrko et al. [23], (see [1,8,9, 11,29] for recent advances and more references on this convergence), statistical convergence of order α by Bhunia et al. [2] (statistical convergence of order α was also independently introduced by Colak [4], more investigations in this direction and more applications can be found in [5]), lacunary statistical convergence of order α by Sengöl & Et. M [30], pointwise and uniform statistical convergence of order α for sequences of functions by Cinar et al. [3], λ-statistical convergence of order α of sequences of function by Et. M et al. [13], $\mathcal{I}$-statistical and $\mathcal{I}$-lacunary statistical convergence of order α by Savas & Das [10], open covers and selection principles by Das [6,7]. The notion of statistical convergence has applications in different fields of mathematics: in number theory by Erdös & Tenenbaum [12], in statistics and probability theory by Fridy & Khan [17] and Ghosal [20–22], in approximation theory by Gadjiev & Orhan [19], in Hopfield neural network by Martinez et al. [25], in optimization by Pehlivan & Mamedov [26].

In particular in probability theory, if for each positive integer n, a random variable X_n is defined on a given event space S (same for each n) with respect to a given class of events $\triangle$ and a probability function $P : \triangle \to \mathbb{R}$ (where $\mathbb{R}$ denotes the set of real numbers) then we say that $X_1, X_2, X_3, \ldots, X_n, \ldots$ is a sequence of random variables and as in analysis we denote this sequence by $\{X_n\}_{n\in\mathbb{N}}$.

From the practical point of view the discussion of a random variable X is highly significant if it is known that there exists a real constant c for which $P(|X - c| < \epsilon) \simeq 1$, where $\epsilon > 0$ is sufficiently small, that is, it is nearly certain that values of X lie in a very small neighbourhood of c.

For a sequence of random variables $\{X_n\}_{n\in\mathbb{N}}$, each X_n may not have the above property but it may happen that the aforementioned property (with respect to a real constant c) becomes more and more distinguishable as n gradually increases and the question of existence of such a real constant c can be answered by a concept of convergence in probability of the sequence $\{X_n\}_{n\in\mathbb{N}}$.

In this short paper we shall limit our discussion to four types of convergence of a sequence of random variables, namely,

(i) statistical convergence of order α in probability,

(ii) strong p-Cesàro summability of order α in probability,

(iii) lacunary statistical convergence or S_θ-convergence of order α in probability,

(iv) N_θ-convergence of order α in probability.

Our main aim in this paper is to establish some important theorems related to the modes of convergence (i)–(iv), which effectively extend and improve all the existing results in