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Hyper-order and fixed points of meromorphic solutions of higher order linear differential equations

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Abstract. The purpose of this paper is to study the growth and fixed points of meromorphic solutions and their derivatives to complex higher order linear differential equations whose coefficients are meromorphic functions. Our results extend the previous results due to Peng and Chen, Xu and Zhang and others.

Keywords: Linear differential equations; Meromorphic solutions; Order of growth; Hyper-order; Fixed points

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1. INTRODUCTION AND STATEMENT OF RESULTS

In this paper, we shall assume that the reader is familiar with the fundamental results and the standard notations of the Nevanlinna value distribution theory of meromorphic functions (see [11,17]). In addition, we will use notations $\sigma(f)$, $\sigma_2(f)$ to denote respectively the order and the hyper-order of growth of a meromorphic function $f(z)$, $\lambda(f)$, $\overline{\lambda}(f)$, $\overline{\tau}(f)$ to denote respectively the exponents of convergence of the zero-sequence, the sequence of distinct zeros and the sequence of distinct fixed points of $f(z)$. See [2,11,14,17] for notations and definitions.

Consider the second order linear differential equation

$$f'' + A_1(z)e^{P(z)}f' + A_0(z)e^{Q(z)}f = 0, \tag{1.1}$$

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where $P(z), Q(z)$ are nonconstant polynomials, $A_1(z)$, $A_0(z)\,(\not\equiv 0)$ are entire functions such that $\sigma(A_1) < \deg P(z)$, $\sigma(A_0) < \deg Q(z)$. Gundersen showed in [9, p. 419] that if $\deg P(z) \neq \deg Q(z)$, then every nonconstant solution of (1.1) is of infinite order. If $\deg P(z) = \deg Q(z)$, then (1.1) may have nonconstant solutions of finite order. For instance $f(z) = e^z + 1$ satisfies $f'' + e^z f' - e^z f = 0$.

In [3], Chen and Shon investigated the case when $\deg P(z) = \deg Q(z)$ and proved the following results.

Theorem A (*[3]*). *Let $A_j(z)\,(\not\equiv 0)\,(j = 0, 1)$ be meromorphic functions with $\sigma(A_j) < 1\,(j = 0, 1)$, a, b be complex numbers such that $ab \neq 0$ and $\arg a \neq \arg b$ or $a = cb$ $(0 < c < 1)$. Then every meromorphic solution $f(z) \not\equiv 0$ of the equation*

$$f'' + A_1(z)\, e^{az} f' + A_0(z)\, e^{bz} f = 0 \tag{1.2}$$

has infinite order.

In the same paper, Chen and Shon investigated the fixed points of solutions, their 1st and 2nd derivatives and the differential polynomials and obtained.

Theorem B (*[3]*). *Let $A_j(z)\,(j = 0, 1), a, b, c$ satisfy the additional hypotheses of Theorem A. Let d_0, d_1, d_2 be complex constants that are not all equal to zero. If $f(z) \not\equiv 0$ is any meromorphic solution of Eq. (1.2), then:*

(i) *f, f', f'' all have infinitely many fixed points and satisfy*

$$\overline{\lambda}(f - z) = \overline{\lambda}(f' - z) = \overline{\lambda}(f'' - z) = \infty,$$

(ii) *the differential polynomial*

$$g(z) = d_2 f'' + d_1 f' + d_0 f$$

has infinitely many fixed points and satisfies $\overline{\lambda}(g - z) = \infty$.

In [13], Peng and Chen investigated the order and hyper-order of solutions of some second order linear differential equations and proved the following result.

Theorem C (*[13]*). *Let $A_j(z)\,(\not\equiv 0)\,(j = 1, 2)$ be entire functions with $\sigma(A_j) < 1$, a_1, a_2 be complex numbers such that $a_1 a_2 \neq 0$, $a_1 \neq a_2$ (suppose that $|a_1| \leqslant |a_2|$). If $\arg a_1 \neq \pi$ or $a_1 < -1$, then every solution $f\,(\not\equiv 0)$ of the differential equation*

$$f'' + e^{-z} f' + \left(A_1 e^{a_1 z} + A_2 e^{a_2 z}\right) f = 0$$

has infinite order and $\sigma_2(f) = 1$.

Recently, Xu and Zhang investigated the order, the hyper-order and fixed points of meromorphic solutions of some second order linear differential equations and proved the following results.

Theorem D (*[16]*). *Suppose that $A_j(z)\,(\not\equiv 0)\,(j = 0, 1, 2)$ are meromorphic functions and $\sigma(A_j) < 1$, and a_1, a_2 are two complex numbers such that $a_1 a_2 \neq 0$, $a_1 \neq a_2$ (suppose*