

Applications of an identity of Andrews

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Abstract. In this paper, we give a bilateral form of an identity of Andrews, which is a generalization of the ${}_1\psi_1$ summation formula of Ramanujan. Using Andrews' identity, we deduce some new identities involving mock theta functions of second order and finally, we deduce some q -gamma, q -beta and eta function identities.

Keywords: q -Series; Mock theta functions; q -Gamma; q -Beta; Eta-functions

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1. INTRODUCTION AND STATEMENT OF RESULTS

In 1981, Andrews [2] has established the following identity

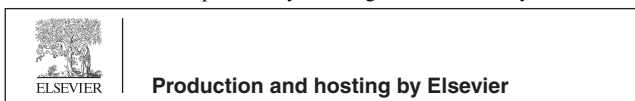
$$\begin{aligned} a^{-1} \sum_{n=0}^{\infty} \frac{(-q/a, AB/ab)_n}{(-B/a, -A/a)_{n+1}} (-b)^n - b^{-1} \sum_{n=0}^{\infty} \frac{(A, -aq/B)_n}{(-a, -A/b)_{n+1}} (-B/b)^n \\ = (a^{-1} - b^{-1}) \frac{(A, B, bq/a, aq/b, q, AB/ab)_{\infty}}{(-b, -a, -A/b, -A/a, -B/b, -B/a)_{\infty}}, \quad |b|, |B/b| < 1, \end{aligned} \quad (1.1)$$

where as usual

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$$\begin{aligned}
 (a)_\infty &:= (a; q)_\infty := \prod_{n=0}^\infty (1 - aq^n), \\
 (a)_n &:= (a; q)_n := \frac{(a)_\infty}{(aq^n)_\infty}, \quad n \text{ is an integer,} \\
 (a_1, a_2, a_3, \dots, a_m)_n &= (a_1)_n (a_2)_n (a_3)_n \cdots (a_m)_n, \\
 (a_1, a_2, a_3, \dots, a_n)_\infty &= (a_1)_\infty (a_2)_\infty (a_3)_\infty \cdots (a_n)_\infty.
 \end{aligned}$$

This identity was proved using several summation and transformation formulae involving basic hypergeometric series. Putting $A = 0$, $a = -q/a$, $B = b/a$ and $b = -z$ in (1.1), we obtain the well-known ${}_1\psi_1$ summation formula of Ramanujan [9].

$$\sum_{n=-\infty}^\infty \frac{(a)_n}{(b)_n} z^n = \frac{(az)_\infty (q)_\infty (q/az)_\infty (b/a)_\infty}{(z)_\infty (b)_\infty (b/az)_\infty (q/a)_\infty}, \tag{1.2}$$

As indicated by Andrews [2] in his paper, Agarwal [1] and Kang [6] have proved (1.1) using the three term transformation formula of ${}_3\phi_2$ -series [3, Equation (III.33), p. 364]. Recently, Liu [7] obtained the following equivalent form of (1.1) using (1.2) along with Roger-Fine identity by employing q -exponential operators.

Theorem 1.3. *If $|a|, |b| < 1$, then*

$$\begin{aligned}
 a^{-1} \sum_{k=0}^\infty \frac{(-q/a, cd/ab)_k}{(-c/a, -d/a)_{k+1}} (-b)^k - b^{-1} \sum_{k=0}^\infty \frac{(-q/b, cd/ab)_k}{(-c/b, -d/b)_{k+1}} (-a)^k \\
 = (a^{-1} - b^{-1}) \frac{(q, aq/b, bq/a, c, d, cd/ab)_\infty}{(-a, -b, -c/a, -c/b, -d/a, -d/b)_\infty}. \tag{1.3}
 \end{aligned}$$

One can recover (1.1) from (1.3) by using Sears transformation for ${}_3\phi_2$ -series [3, Equation (III.9), p. 359].

The main objective of this paper is to give a bilateral form of (1.3). As applications of (1.3) we derive some new identities involving mock theta functions of second order and also some q -gamma, q -beta and eta-function identities.

The q -gamma function $\Gamma_q(x)$, was introduced by Thomae [11] and later by Jackson [5] as

$$\Gamma_q(x) = \frac{(q)_\infty}{(q^x)_\infty} (1 - q)^{1-x}, \quad 0 < q < 1. \tag{1.4}$$

q -Beta function is defined by

$$B_q(x, y) = (1 - q) \sum_{n=0}^\infty \frac{(q^{n+1})_\infty}{(q^{n+y})_\infty} q^{nx}.$$

A relation between q -Beta function and q -gamma function is given by

$$B_q(x, y) = \frac{\Gamma_q(x)\Gamma_q(y)}{\Gamma_q(x+y)}. \tag{1.5}$$

The Dedekind eta function is defined by

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