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Implicit iterative method for approximating a common solution of split equilibrium problem and fixed point problem for a nonexpansive semigroup

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Abstract. In this paper, we introduce and study an implicit iterative method to approximate a common solution of split equilibrium problem and fixed point problem for a nonexpansive semigroup in real Hilbert spaces. Further, we prove that the nets generated by the implicit iterative method converge strongly to the common solution of split equilibrium problem and fixed point problem for a nonexpansive semigroup. This common solution is the unique solution of a variational inequality problem and is the optimality condition for a minimization problem. Furthermore, we justify our main result through a numerical example. The results presented in this paper extend and generalize the corresponding results given by Plubtieng and Punpaeng [S. Plubtieng, R. Punpaeng, Fixed point solutions of variational inequalities for nonexpansive semigroups in Hilbert spaces, Math. Comput. Model. 48 (2008) 279–286] and Cianciaruso et al. [F. Cianciaruso, G. Marino, L. Muglia, Iterative methods for equilibrium and fixed point problems for nonexpansive semigroups in Hilbert space, J. Optim. Theory Appl. 146 (2010) 491–509].

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1. Introduction

Throughout the paper unless otherwise stated, let H_1 and H_2 be real Hilbert spaces with inner product $\langle \cdot , \cdot \rangle$ and norm $\| \cdot \|$. Let C and Q be nonempty closed convex subsets of H_1 and H_2, respectively.

In 1994, Blum and Oettli [2] introduced and studied the following equilibrium problem (in short, EP): Find $x \in C$ such that

$$F(x, y) \geqslant 0, \quad \forall y \in C, \tag{1.1}$$

where $F : C \times C \to \mathbb{R}$ is a bifunction.

The EP (1.1) includes variational inequality problems, optimization problems, Nash equilibrium problems, saddle point problems, fixed point problems, and complementary problems as special cases. In other words, EP (1.1) is an unify model for several problems arising in science, engineering, optimization, economics, etc.

In the last two decades, EP (1.1) has been generalized and extensively studied in many directions due to its importance; see, for example [14,16–19] and references therein, for the literature on the existence of solution of the various generalizations of EP (1.1). Some iterative methods have been studied for solving various classes of equilibrium problems, see for example [4,10,13,20–23,30,31] and references therein. Recently, some iterative methods for finding a common solution for system of equilibrium problems have been studied in the same space, see for example [9,28]. In general, the equilibrium problems in systems lie in the different spaces. Therefore, in this paper, we consider the following pair of equilibrium problems in different spaces, which is called *split equilibrium problem* (in short, SEP) due to Moudafi [25]:

Let $F_1 : C \times C \to \mathbb{R}$ and $F_2 : Q \times Q \to \mathbb{R}$ be nonlinear bifunctions and $A : H_1 \to H_2$ be a bounded linear operator, then the *split equilibrium problem* (SEP) is to find $x^* \in C$ such that

$$F_1(x^*, x) \geqslant 0, \quad \forall x \in C \tag{1.2}$$

and such that

$$y^* = Ax^* \in Q \text{ solves } F_2(y^*, y) \geqslant 0, \quad \forall y \in Q. \tag{1.3}$$

When looked separately, (1.2) is the equilibrium problem (EP) and we denoted its solution set by EP(F_1). The SEP (1.2) and (1.3) constitutes a pair of equilibrium problems which have to be solved so that the image $y^* = Ax^*$ under a given bounded linear operator A, of the solution x^* of the EP (1.2) in H_1 is the solution of another EP (1.3) in another space H_2, we denote the solution set of EP (1.3) by EP(F_2). The solution set of SEP (1.2) and (1.3) is denoted by $\Omega = \{p \in \text{EP}(F_1) : Ap \in \text{EP}(F_2)\}$.

SEP (1.2) and (1.3) generalize a multiple-set split feasibility problem. It also includes as special case, the split variational inequality problem [7] which is the generalization of split zero problems and split feasibility problems, see for detail [3,5–7,25,26].

Example 1.1. Let $H_1 = H_2 = \mathbb{R}$, the set of all real numbers, with the inner product defined by $\langle x, y \rangle = xy, \forall x, y \in \mathbb{R}$. Let $C = [0,2]$ and $Q = (-\infty,0]$; let $F_1 : C \times C \to \mathbb{R}$ and $F_2 : Q \times Q \to \mathbb{R}$ be defined by $F_1(x,y) = x^2 - y, \forall x,y \in C$ and $F_2(u,v) = (u + 6)(v - u), \forall\ u,v \in Q$ and let, for each $x \in \mathbb{R}$, we define $A(x) = -3x$. It is easy to