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Generating relations of multi-variable Tricomi functions of two indices using Lie algebra representation

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Abstract. This paper is an attempt to stress the usefulness of the multi-variable special functions. In this paper, we derive certain generating relations involving 2-indices 5-variables 5-parameters Tricomi functions (2I5V5PTF) by using a Lie-algebraic method. Further, we derive certain new and known generating relations involving other forms of Tricomi and Bessel functions as applications.

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1. Introduction

The function [1]

$$C_n(x) = \sum_{r=0}^{\infty} \frac{(-1)^r \ x^r}{r!(n+r)!}, \tag{1.1}$$

is a Bessel like function known as Tricomi function and is related to the cylindrical Bessel function $J_n(x)$ by the following link [1]:

$$C_n(x) = x^{-n/2} J_n\left(2\sqrt{x}\right). \tag{1.2}$$

The Tricomi function $C_n(x)$ is defined by means of the generating function

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$$\exp\left(t-\frac{x}{t}\right)=\sum_{n=-\infty}^{\infty}C_n(x)\ t^n, \tag{1.3}$$

which yields the recurrence relations

$$\frac{d}{dx}\ C_n(x)=-C_{n+1}(x), \tag{1.4a}$$

$$x\ C_{n+1}(x)-nC_n(x)+C_{n-1}(x)=0. \tag{1.4b}$$

On combining the above recurrences, we get the following differential equation satisfied by $C_n(x)$:

$$\left(x\ \frac{d^2}{dx^2}+(n+1)\frac{d}{dx}+1\right)C_n(x)=0. \tag{1.5}$$

The study of the properties of multi-variable generalized special functions has provided new means of analysis for the derivation of the solution of large classes of partial differential equations often encountered in physical problems. The relevance of the special functions in physics is well established. Most of the special functions of mathematical physics as well as their generalizations have been suggested by physical problems.

In order to further stress the usefulness of the generalized special functions, Dattoli et al. [3] have introduced the three variables two indices extension of Tricomi functions defined as:

$$C_{m,n}(x,y,z)=\sum_{s=0}^{\infty}C_{m+s}(x)\ C_{n+s}(y)\ \frac{z^s}{s!}. \tag{1.6}$$

The generating function for 2-indices 3-variables Tricomi functions (2I3VTF) $C_{m,n}(x,y,z)$ is given as:

$$\exp\left(\left(u-\frac{x}{u}\right)+\left(v-\frac{y}{v}\right)+\frac{z}{uv}\right)=\sum_{m,n=-\infty}^{\infty}C_{m,n}(x,y,z)u^m\ v^n. \tag{1.7}$$

We call $\boldsymbol{x}=(x_1,x_2,x_3,x_4,x_5)$ and $\boldsymbol{\xi}=(\xi_1,\xi_2,\xi_3,\xi_4,\xi_5)$, where $\xi_1,\ \xi_2,\ \xi_3,\ \xi_4$ and ξ_5 are arbitrary complex parameters.

For more further generalizations, along with the function one index-two variables Tricomi function defined by the generating function [2]

$$\exp\left(t-\frac{x}{t}+t^2-\frac{y}{t^2}\right)=\sum_{n=-\infty}^{\infty}C_n(x,y)\ t^n, \tag{1.8}$$

and the expansion series:

$$C_n(x,y)=\sum_{l=-\infty}^{\infty}C_{n-2l}(x)\ C_l(y), \tag{1.9}$$

we can introduce the five variables two indices extension of Tricomi functions defined as:

$$C_{m,n}(\boldsymbol{x})=\sum_{s=0}^{\infty}C_{m+s}(x_1,x_3)\ C_{n+s}(x_2,x_4)\ \frac{x_5^s}{s!}. \tag{1.10}$$