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Vector implicit quasi complementarity problems

Suhel Ahmad Khan [a,*], Byung Soo Lee [b]

[a] Department of Mathematics, BITS-PILANI, Dubai Campus, Dubai, United Arab Emirates

[b] Department of Mathematics, Kyungsung University, Busan 608-736, Republic of Korea

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Abstract. In this work, we establish some existence theorems for solutions to a new class of vector implicit quasi complementarity problems and the corresponding vector implicit quasi variational inequality problems. Further we introduce the notion of a local non-positivity of a pair of mappings (F, Q) and consider the existence and properties of solutions for vector implicit quasi variational inequality problems and the corresponding vector implicit quasi complementarity problems in the neighborhood of a given point belonging to an underlined domain K.

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1. Introduction

The complementarity problem theory was introduced and studied by Lemke [19] and Cottle and Dantzig [3] in 1960. Also, in the 1960s, variational inequality was introduced by Hartman and Stampacchia [9] and Browder [1]. In 1971, Karamardian [14] firstly considered the equivalence of some scalar complementarity problems with solution sets $C(F,K) = \{x \in K\colon F(x) \in K^{\star}, \langle x, F(x)\rangle = 0\}$ and some scalar variational type problems with solution sets $V(F,K) = \{x \in K\colon \langle u - x, F(x)\rangle \geqslant 0 \text{ for all } u \in K\}$ for a mapping F defined on a closed convex cone K in a locally convex Hausdorff topological vector space X to a vector space Y. Since then, there have been much research

* Corresponding author.
E-mail addresses: khan.math@gmail.com (S.A. Khan), bslee@ks.ac.kr (B.S. Lee).
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[2,4,6–8,11,12,15–18,20–24] on the equivalence between a range of many kinds of complementarity problems and corresponding variational inequality problems under suitable different conditions. Solutions of these class of problems have extensive and important applications in vector optimization, optimal control, mathematical programing, operations research, and equilibrium problem of economics. Inspired and motivated by the above applications, various generalized variational inequality problems, the generalized complementarity problems have become important developed directions of variational inequality theory (for example, see [3,4,14]).

In 2001, Yin et al. [25] introduced a class of F-complementarity problems, which consist in finding $x \in K$ such that

$$\langle Tx, x\rangle + F(x) = 0, \quad \langle Tx, y\rangle + F(y) \geqslant 0, \quad \forall y \in K,$$

where X is a Banach space with topological dual X^*, and $\langle .,. \rangle$ duality pairing between them, K a closed convex cone of X, and T: $K \to X^*$, $F : K \to \mathbb{R}$. They obtained an existence theorem for solving F-complementarity problems and also proved that if F is positively homogeneous (i.e. $F(tx) = tF(x)$ for all $t > 0$ and $x \in K$), the F-complementarity problem is equivalent to the following generalized variational inequality problem which consists in finding $x \in K$ such that

$$\langle Tx, y - x\rangle + F(y) - F(x) \geqslant 0, \quad \forall y \in K.$$

In 2003, Fang and Huang [6] introduced a class of vector F-complementarity problems and investigated the solvability of the class for demipseudomonotone mappings and finite-dimensional continuous mappings in reflexive Banach spaces. Later, Huang and Li [12] introduced a class of scalar F-implicit complementarity problems and the corresponding variational inequality problems in Banach spaces. In 2006, Li and Huang [20] extended the result in [12] to the vector case and presented the equivalence between the vector F-implicit complementarity problems and the corresponding vector F-implicit variational inequality problems. They obtained some existence theorems for solutions for their problems.

Recently, Lee et al. [18] extended Li and Huang's results in the setting of set-valued mapping. They studied a class of vector F-implicit complementarity problems and established some existence results in topological vector spaces without considering the continuity or the monotonicity on mappings. Recently, Wu and Huang [22] introduced a class of mixed vector F-implicit complementarity problems and the corresponding mixed vector F-implicit variational inequality problems. They derived some existence theorems of solutions for the mixed vector F-implicit complementarity problems and the mixed vector F-implicit variational inequality problems by using Fan-KKM theorem under some suitable assumptions without the monotonicity in the neighborhood of a given point belonging to an underlined domain K of the set-valued mappings, where the neighborhood is contained in K.

Very recently, Khan [16] introduced and studied the following vector implicit quasi complementarity problem of finding $x \in K$ such that

$$\langle N(Ax, Tx), g(x)\rangle + F(g(x), g(x)) = 0$$

and

$$\langle N(Ax, Tx), h(y)\rangle + F(h(y), g(x)) \in P(x), \quad \forall y \in K,$$