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On a multi point boundary value problem for a fractional order differential inclusion

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Abstract. The existence of solutions for a multi point boundary value problem of a fractional order differential inclusion is investigated. Several results are obtained by using suitable fixed point theorems when the right hand side has convex or non convex values.

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1. Introduction

Differential equations with fractional order have recently proved to be strong tools in the modeling of many physical phenomena; for a good bibliography on this topic we refer to [18]. As a consequence there was an intensive development of the theory of differential equations of fractional order [2,16,22] etc.. The study of fractional differential inclusions was initiated by El-Sayed and Ibrahim [13]. Very recently several qualitative results for fractional differential inclusions were obtained in [1,3,6–11,15,20] etc.

In this paper we study the following problem

$$D^{\alpha}x(t) \in F(t, x(t), x'(t)) \quad a.e.\ [0, 1], \tag{1.1}$$

$$x(0) = x'(0) = 0, \quad x(1) - \sum_{i=1}^{m} a_i x(\xi_i) = \lambda, \tag{1.2}$$

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where D^α is the standard Riemann–Liouville fractional derivative, $\alpha \in (2,3]$, $m \geqslant 1$, $0 < \xi_1 < \cdots < \xi_m < 1$, $\sum_{i=1}^{m} a_i \xi_i^{\alpha-1} < 1$, $\lambda > 0$, $a_i > 0$, $i = \overline{1,m}$ and $F : [0, 1] \times \mathbf{R} \times \mathbf{R} \to \mathcal{P}(\mathbf{R})$ is a set-valued map.

The present paper is motivated by a recent paper of Nyamoradi [19], where it is considered problem (1.1) and (1.2) with F single valued and several existence results are provided.

The aim of our paper is to extend the study in [19] to the set-valued framework and to present some existence results for problem (1.1) and (1.2). Our results are essentially based on a nonlinear alternative of Leray–Schauder type, on Bressan–Colombo selection theorem for lower semicontinuous set-valued maps with decomposable values and on Covitz and Nadler set-valued contraction principle. The methods used are known ([1,8,9] etc.), however their exposition in the framework of problem (1.1) and (1.2) is new.

The paper is organized as follows: in Section 2 we recall some preliminary facts that we need in the sequel and in Section 3 we prove our main results.

2. Preliminaries

In this section we sum up some basic facts that we are going to use later.

Let (X,d) be a metric space with the corresponding norm $|\cdot|$ and let $I \subset \mathbf{R}$ be a compact interval. Denoted by $\mathcal{L}(I)$ the σ-algebra of all Lebesgue measurable subsets of I, by $\mathcal{P}(X)$ the family of all nonempty subsets of X and by $\mathcal{B}(X)$ the family of all Borel subsets of X. If $A \subset I$ then $\chi_A: I \to \{0,1\}$ denotes the characteristic function of A. For any subset $A \subset X$ we denote by $\overline{A}$ the closure of A.

Recall that the Pompeiu–Hausdorff distance of the closed subsets $A,B \subset X$ is defined by

$$d_H(A,B) = \max\{d^*(A,B), d^*(B,A)\}, \quad d^*(A,B) = \sup\{d(a,B); a \in A\},$$

where $d(x,B) = \inf_{y \in B} d(x,y)$.

As usual, we denote by $C(I,X)$ the Banach space of all continuous functions $x: I \to X$ endowed with the norm $|x|_C = \sup_{t \in I} |x(t)|$ and by $L^1(I,X)$ the Banach space of all (Bochner) integrable functions $x: I \to X$ endowed with the norm $|x|_1 = \int_I |x(t)| \mathrm{d}t$.

A subset $D \subset L^1(I,X)$ is said to be *decomposable* if for any $u,v \in D$ and any subset $A \in \mathcal{L}(I)$ one has $u\chi_A + v\chi_B \in D$, where $B = I \backslash A$.

Consider $T : X \to \mathcal{P}(X)$ a set-valued map. A point $x \in X$ is called a fixed point for T if $x \in T(x)$. T is said to be bounded on bounded sets if $T(B) := \cup_{x \in B} T(x)$ is a bounded subset of X for all bounded sets B in X. T is said to be compact if $T(B)$ is relatively compact for any bounded sets B in X. T is said to be totally compact if $\overline{T(X)}$ is a compact subset of X. T is said to be upper semicontinuous if for any open set $D \subset X$, the set $\{x \in X: T(x) \subset D\}$ is open in X. T is called completely continuous if it is upper semicontinuous and totally bounded on X.

It is well known that a compact set-valued map T with nonempty compact values is upper semicontinuous if and only if T has a closed graph.

We recall the following nonlinear alternative of Leray–Schauder type and its consequences.

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