

## A remark on the existence of positive solutions for variable exponent elliptic systems

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**Abstract.** In this article, we consider the system of differential equations

$$\begin{cases} -\Delta_{p(x)} u = \lambda^{p(x)} [a(x)u^{\alpha(x)} v^{\gamma(x)} + h_1(x)] & \text{in } \Omega, \\ -\Delta_{q(x)} v = \lambda^{q(x)} [b(x)u^{\beta(x)} v^{\delta(x)} + h_2(x)] & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega, \end{cases}$$

where  $\Omega \subset \mathbb{R}^N$  is a bounded domain with  $C^2$ -boundary  $\partial\Omega$ ,  $1 < p(x)$ ,  $q(x) \in C^1(\overline{\Omega})$  are functions. The operator  $-\Delta_{p(x)} u = -\operatorname{div}(|\nabla u|^{p(x)-2} \nabla u)$  is called the  $p(x)$ -Laplacian. When  $\alpha$ ,  $\beta$ ,  $\delta$ ,  $\gamma$  satisfy some suitable conditions, we prove the existence of positive solution via sub-supersolution arguments without assuming sign conditions on the functions  $h_1$  and  $h_2$ .

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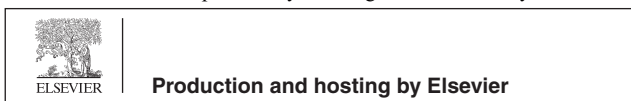
### 1. INTRODUCTION AND PRELIMINARIES

The study of differential equations and variational problems with nonstandard  $p(x)$ -growth conditions is a new and interesting topic. It arises from nonlinear elasticity theory, electrorheological fluids, etc. (see [1,2,14,20]). Many existence results have been obtained on this kind of problems, see for example [4,9,10,12,15–18]. In [6–8], Fan

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et al. studied the regularity of solutions for differential equations with nonstandard  $p(x)$ -growth conditions.

In this paper, we mainly consider the existence of positive weak solutions for the system

$$\begin{cases} -\Delta_{p(x)}u = \lambda^{p(x)}[a(x)u^{\alpha(x)}v^{\gamma(x)} + h_1(x)] & \text{in } \Omega, \\ -\Delta_{q(x)}v = \lambda^{q(x)}[b(x)u^{\delta(x)}v^{\beta(x)} + h_2(x)] & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where  $\Omega \subset \mathbb{R}^N$  is a bounded domain with  $C^2$ -boundary  $\partial\Omega$ ,  $1 < p(x), q(x) \in C^1(\overline{\Omega})$  are two functions. The operator  $-\Delta_{p(x)}u = -\operatorname{div}(|\nabla u|^{p(x)-2}\nabla u)$  is called the  $p(x)$ -Laplacian and the corresponding equation is called a variable exponent equation. Especially, if  $p(x) \equiv p$  (a constant), (1.1) is the well-known  $(p, q)$ -Laplacian system and the corresponding equation is called a constant exponent equation. We have known that the existence of solutions for  $p$ -Laplacian elliptic systems has been intensively studied in the last decades, we refer to [3,11,13]. In [11], Hai et al. considered the existence of positive weak solutions for the  $p$ -Laplacian problem

$$\begin{cases} -\Delta_p u = \lambda f(v) & \text{in } \Omega, \\ -\Delta_p v = \lambda g(u) & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.2}$$

in which the first eigenfunction was used for constructing the subsolution of  $p$ -Laplacian problems. Under the condition

$$\lim_{u \rightarrow +\infty} \frac{f(M(g(u))^{1/(p-1)})}{u^{p-1}} = 0 \text{ for all } M > 0,$$

the authors showed that the problem (1.2) has at least one positive solution provided that  $\lambda > 0$  is large enough.

In [3], the author studied the existence and nonexistence of positive weak solution to the following quasilinear elliptic system

$$\begin{cases} -\Delta_p u = \lambda f(u, v) = \lambda u^\alpha v^\gamma & \text{in } \Omega, \\ -\Delta_q v = \lambda g(u, v) = \lambda u^\delta v^\beta & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega. \end{cases} \tag{1.3}$$

The first eigenfunction is used to construct the subsolution of problem (1.3), the main results are as follows:

- (i) If  $\alpha, \beta \geq 0, \gamma, \delta > 0$ ,  $\theta = (p - 1 - \alpha)(q - 1 - \beta) - \gamma\delta > 0$ , then problem (1.3) has a positive weak solution for each  $\lambda > 0$ ;
- (ii) If  $\theta = 0$  and  $p\gamma = q(p - 1 - \alpha)$ , then there exists  $\lambda_0 > 0$  such that for  $0 < \lambda < \lambda_0$ , then problem (1.3) has no nontrivial nonnegative weak solution.

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