Arab Journal of Mathematical Sciences (2012) **18**, 25–37

King Saud University
Arab Journal of Mathematical Sciences
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ORIGINAL ARTICLE

Haar wavelet method for solving generalized Burgers–Huxley equation

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Received 3 February 2011; revised 18 July 2011; accepted 6 August 2011
Available online 14 October 2011

KEYWORDS
Haar wavelets;
Nonlinear PDE;
Generalized Burgers–Huxley equation;
Approximate solution

Abstract In this paper, an efficient numerical method for the solution of nonlinear partial differential equations based on the Haar wavelets approach is proposed, and tested in the case of generalized Burgers–Huxley equation. Approximate solutions of the generalized Burgers–Huxley equation are compared with exact solutions. The proposed scheme can be used in a wide class of nonlinear reaction–diffusion equations. These calculations demonstrate that the accuracy of the Haar wavelet solutions is quite high even in the case of a small number of grid points. The present method is a very reliable, simple, small computation costs, flexible, and convenient alternative method.

1. Introduction

In this paper, following approximate solutions of the following nonlinear diffusion equation is considered:

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Peer review under responsibility of King Saud University.
doi:10.1016/j.ajmsc.2011.08.003

Production and hosting by Elsevier

$$\frac{\partial u}{\partial t} + \alpha u^{\delta}\frac{\partial u}{\partial x} - \frac{\partial^2 u}{\partial x^2} = \beta u(1 - u^{\delta})(u^{\delta} - \gamma) \tag{1}$$

by the Haar wavelet method. Where α, β, γ and δ are parameters, $\beta \geqslant 0$, $\gamma > 0$, $\gamma \in (0,1)$. Eq. (1) is a generalized Burgers–Huxley equation. When $\alpha = 0$, $\delta = 1$, Eq. (1) is reduced to the Huxley equation which describes nerve pulse propagation in nerve fibers and wall motion in liquid crystals [22,23]. When $\beta = 0$, $\delta = 1$, Eq. (1) is reduced to the Burgers equation describing the far field of wave propagation in nonlinear dissipative systems [26]. When $\alpha = 0$, $\beta = 1$, $\delta = 1$, Eq. (1) becomes the FitzHugh–Nagumo (FN) equation which is a reaction–diffusion equation used in circuit theory, biology and the area of population genetics [1]. At $\delta = 1$ and $\alpha \neq 0$, $\beta \neq 0$, Eq. (1) is turned into the Burgers–Huxley equation. This equation, which shows a prototype model for describing the interaction between reaction mechanisms, convection effects and diffusion transport, was investigated by Satsuma [21].

Various numerical techniques were used in the literature to obtain numerical solutions of the Burgers–Huxley equation. Wang et al. [24] studied the solitary wave solution of the generalized Burgers–Huxley equation while Estevez [7] presented nonclassical symmetries and the singular modified Burgers and Burgers–Huxley equation. Also Estevez and Gordoa [8] applied a complete Painleve test to the generalized Burgers–Huxley equation. In the past few years, various mathematical methods such as spectral methods [5,14,15], Adomian decomposition method [11–13], homotopy analysis method [19], the tanh-coth method [25], variational iteration method [2,3], Hopf-Cole transformation [6] and polynomial differential quadrature method [20] have been used to solve the equation.

In solving ordinary differential equations (ODEs), Chen and Hsiao [4] derived an operational matrix of integration based on the Haar wavelet method. By using the Haar wavelet method, Lepik [16], Lepik [17] solved higher order as well as nonlinear ODEs and some nonlinear evolution equations. Lepik [18] also used this method to solve Burgers and sine-Gordon equations. Hariharan et al. [10], Hariharan and Kannan [9] introduced the Haar wavelet method for solving both Fisher's and FitzHugh–Nagumo equations.

In the present paper, a new direct computational method for solving generalized Burgers–Huxley equation is introduced. This method consists of reducing the problem to a set of algebraic equation by first expanding the term, which has maximum derivative, given in the equation as Haar functions with unknown coefficients. The operational matrix of integration and product operational matrix are utilized to evaluate the coefficients of the Haar functions. Identification and optimization procedures of the solutions are greatly reduced or simplified. Since the integration of the Haar functions vector is a continuous function, the solutions obtained are continuous. This method gives us the implicit form of the approximate solutions of the problems. In this method, a few sparse matrixes can be obtained, and there are no complex integrals or methodology. Therefore, the present method is useful for obtaining the implicit form of the approximations of linear or