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ORIGINAL ARTICLE

Existence of solutions for fractional differential inclusions with nonlocal strip conditions

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Abstract In this paper, we discus the existence of solutions for a nonlocal boundary value problem of fractional differential inclusions concerning a nonlocal strip condition via some fixed point theorems. Our results include the cases when the right-hand side of the inclusion is convex as well as nonconvex valued.

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1. Introduction

In this paper, we discuss the existence of solutions for a boundary value problem of nonlinear fractional differential inclusions of order $q \in (1,2)$ with nonlocal strip conditions given by

$$\begin{cases} {}^cD^q x(t) \in F(t, x(t)), \ 0 < t < 1, 1 < q \leqslant 2, \\ x(0) = 0, \ x(1) = \eta \int_v^\tau x(s)ds, \ 0 < v < \tau < 1 (v \neq \tau), \end{cases} \tag{1.1}$$

where ${}^c\mathrm{D}^q$ denotes the Caputo fractional derivative of order q, $F : [0, 1] \times \mathbb{R} \to \mathcal{P}(\mathbb{R})$ is a multivalued map, $\mathcal{P}(\mathbb{R})$ is the family of all subsets of $\mathbb{R}$.

The single-valued problem, that is the equation ${}^c\mathrm{D}^q x(t) = f(t, x(t))$, with the boundary conditions in (1.1) was studied recently in [4]. As argued in [4], the nonlocal strip condition ($x(1) = \eta \int_v^\tau x(s)ds, \ 0 < v < \tau < 1$) in (1.1) is an extension of a three-point nonlocal boundary condition of the form $x(1) = \eta x(v)$, $\eta \in \mathbb{R}$, $0 < v < 1$. In fact, this strip condition corresponds to a continuous distribution of the values of the unknown function on an arbitrary finite segment of the interval. In other words, the strip condition in (1.1) can be regarded as a four-point nonlocal boundary condition which reduces to the typical integral boundary conditions in the limit $v \to 0$, $\tau \to 1$. Strip conditions of fixed size appear in the mathematical modeling of real world problems, for example, see [6,12]. Thus, the present idea of nonlocal strip conditions will be quite fruitful in modeling the strip problems as one can choose a strip of arbitrary size according to the requirement by fixing the nonlocal parameters involved in the problem. As a matter of fact, integral boundary conditions have various applications in applied fields such as blood flow problems, chemical engineering, thermo-elasticity, underground water flow, population dynamics, etc. For a detailed description of the integral boundary conditions, we refer the reader to the papers [5,16] and references therein. For the basic theory of fractional differential equations and its applications see [24–27], and the recent development on the topic can be found in [1,2,10,3,7–9,11,13,14] and the references cited therein.

Here we extend the results of [4] to cover the multi-valued case. We establish the existence of results for the problem (1.1), when the right hand side is convex as well as nonconvex valued. The first result relies on the nonlinear alternative of Leray–Schauder type. In the second result, we shall combine the nonlinear alternative of Leray–Schauder type for single-valued maps with a selection theorem due to Bressan and Colombo for lower semicontinuous multivalued maps with nonempty closed and decomposable values, while in the third result, we shall use the fixed point theorem for contraction multivalued maps due to Covitz and Nadler.

The methods used are standard, however their exposition in the framework of problems (1.1) is new.