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Numerical simulations for the pricing of options in jump diffusion markets

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Abstract In this paper we find numerical solutions for the pricing problem in jump diffusion markets. We utilize a model in which the underlying asset price is generated by a process that consists of a Brownian motion and an independent compensated Poisson process. By risk neutral pricing the option price can be expressed as an expectation. We simulate the option price numerically using the Monte Carlo method.

1. Introduction

Options are financial derivative products that give the right, but not the obligation, to engage in a future transaction on some underlying financial instrument. For instance, a European call option on a financial underlying asset S-with price $(S_t)_{t\in[0,T]}$ is a contract between two agents (buyer and seller) which gives the holder the right to

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buy S at a pre-specified future time T (the expiration date) for an amount K (called the strike). Moreover, at the signature of the contract ($t = 0$) the buyer pays an amount of money called the premium and at the expiration date ($t = T$) he obtains the payoff $h(S_T) = \max(S_T - K, 0) = (S_T - K)^+$. Naturally, two questions arise herein: (i) what price should the seller charge for the option? (known as the pricing problem); (ii) which self-financing strategy should the seller use to secure a wealth equals to the payoff at $t = T$? (known as the hedging problem).

Most of the works on modeling financial derivatives assume that the underlying asset prices S is a continuous process. For instance, in the pioneer work of Black and Scholes (1973) financial asset prices are modeled by the Brownian motion. One of the shortcoming of this model is that, it does not consider the random jumps which can occur in the prices at any time. However, the international financial crisis has shown the importance of adding jumps to financial modeling for stock prices. Unlike the continuous case, models with jumps allow for the possibility that at any moment, a financial price can suddenly decrease (or increase) and attain a significant lower (or higher) value in a negligible time.

Indeed, many researchers have studied financial models with jumps (Bellamy and Jeanblanc, 2000; Dritschel and Protter, 1999; El-Khatib and Privault, 2003; Jeanblanc and Privault, 2002; Merton, 1976), but the issue has not been resolved because of some theoretical complications. Thus, these models generate incomplete markets where the contingent claim (payoff) can impossibly be hedged.

In this paper we study the pricing problem for an underlying asset price with jumps which is governed by the following stochastic differential equation:

$$\frac{dS_t}{S_t} = r_t dt + \sigma_t[a_t dW_t + b_t d(N_t - \lambda_t t)], \quad t \in [0, T], \quad S_0 \text{ is given} > 0, \qquad (1.1)$$

where r, σ, a, b are deterministic functions such that $1 + \sigma_t b_t > 0$. Here $(N_t)_{t\in[0,T]}$ is a Poisson process with deterministic intensity λ and $(W_t)_{t\in[0,T]}$ is a Brownian motion. Note that the process M defined by $M_t := N_t - \lambda_t t$ for $t \in [0,T]$ is the compensated process associated to N. We consider a market with two assets: the risky asset S given by the Eq. (1.1) to which is related a European call option and a risk-free asset given by

$$dA_t = r_t A_t dt, \quad t \in [0, T], \quad A_0 = 1.$$

We work on a probability space $(\Omega, \mathcal{F}, P)$. $(M_t)_{t\in[0,T]}$ and $(W_t)_{t\in[0,T]}$ are independent and we denote by $(\mathcal{F}_t)_{t\in[0,T]}$ the filtration generated by $(N_t)_{t\in[0,T]}$ and $(W_t)_{t\in[0,T]}$. We assume that (1.1) is the price of the asset under the risk-neutral probability P. Recall that a stochastic process is a function of two variables the time $t \in [0, T]$ and the event $\omega \in \Omega$, but in the literature it is common to write S_t, while it means $S_t := S_t(\omega)$. The same interpretation is true for W_t, N_t and M_t or any other stochastic process in this paper. To the authors knowledge, it is impossible to find an explicit formula for the solution of the pricing problem. However, the premium can be determined and expressed in the following expectation form (see Harrison and Kreps, 1979; Harrison and Pliska, 1981)