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Bifurcation of limit cycles from a non-smooth perturbation of a two-dimensional isochronous cylinder

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ABSTRACT

Detect the birth of limit cycles in non-smooth vector fields is a very important matter into the recent theory of dynamical systems and applied sciences. The goal of this paper is to study the bifurcation of limit cycles from a continuum of periodic orbits filling up a two-dimensional isochronous cylinder of a vector field in $\mathbb{R}^3$. The approach involves the regularization process of non-smooth vector fields and a method based in the Malkin bifurcation function for C^0 perturbations. The results provide sufficient conditions in order to obtain limit cycles emerging from the cylinder through smooth and non-smooth perturbations of it. To the best of our knowledge they also illustrate the implementation by the first time of a new method based in the Malkin bifurcation function. In addition, some points concerning the number of limit cycles bifurcating from non-smooth perturbations compared with smooth ones are studied. In summary the results yield a better knowledge about limit cycles in non-smooth vector fields in $\mathbb{R}^3$ and

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explicit a manner to obtain them by performing non-smooth perturbations in codimension one Euclidean manifolds.

1. Introduction

1.1. Setting the problem

Non-smooth vector fields have become certainly one of the common frontiers between Mathematics and Physics or Engineering. Many authors have contributed to the study of non-smooth vector fields (see for instance the pioneering work [1] or the didactic works [2,3], and the references therein about details of these multi-valued vector fields). In our approach Filippov's convention is considered, see [1]. So, the vector field of the model is non-smooth across a *switching manifold* and it is possible for its trajectories to be confined onto the switching manifold itself. The occurrence of such behavior, known as *sliding motion*, has been reported in a wide range of applications. We can find important examples in electrical circuits having switches, in mechanical devices in which components collide into each other, in problems with friction, sliding or squealing, among others (see [2]).

This work concerns with the existence of limit cycles emerging from a continuum of periodic solutions filling up a two dimensional cylinder via a non-smooth perturbation. Such kind of problems are closed related to the weakest version of the famous 16th Hilbert's problem proposed by Arnol'd (see [4] and [5]). Arnol'd asked about the number of limit cycles bifurcating from the perturbation of a center and up to now many authors have contributed with this subject. However, the problems of perturbation of a submanifold filled up by periodic solutions which appears in the literature are usually restricted to the plane. In our opinion the perturbation of other kind of two-dimensional manifolds has been poorly treated in the literature, and this is the goal of this paper.

Recently in [6] the authors investigated the problem of perturbation of a two-dimensional cylinder filled up by periodic solutions in $\mathbb{R}^3$ by a smooth function. In their paper, the authors illustrated the implementation of a method based in the averaging theory for computing the limit cycles bifurcating from a continuum of periodic solutions occupying a cylinder. Other papers with similar approaches can be found in [7] and [8].

In this paper the goal is to generalize the study presented in [6] for a biggest class of cylinders and also take into account non-smooth perturbations. We stress out that this is not the situation considered in paper [6]. We consider the differential system

$$\begin{aligned} \dot{x} &= -y + x(x^2 + y^2 - 1), \\ \dot{y} &= x + y(x^2 + y^2 - 1), \\ \dot{z} &= h(x, y). \end{aligned} \tag{1}$$