

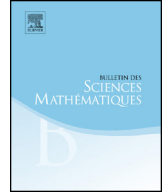


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Serre duality and Hörmander's solution of the $\bar{\partial}$ -equation



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ABSTRACT

We use duality in the manner of Serre to generalize a theorem of Hedenmalm on solution of the $\bar{\partial}$ equation with inverse of the weight in Hörmander L^2 estimates.

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1. Introduction

Let φ be a C^2 strictly sub harmonic function in the complex plane \mathbb{C} , i.e. with Δ the Laplacian, $\Delta\varphi > 0$ in \mathbb{C} . Let $A^2(\mathbb{C}, e^{-2\varphi})$ be the space of all holomorphic functions g in \mathbb{C} such that

$$\|g\|_{L^2(\mathbb{C}, e^{-2\varphi})} := \int_{\mathbb{C}} |g|^2 e^{-2\varphi} dA < \infty,$$

with dA the Lebesgue measure in \mathbb{C} . Suppose that $f \in L^2(\mathbb{C}, e^{2\varphi})$ verifies

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$$\forall g \in A^2(\mathbb{C}, e^{-2\varphi}), \int_{\mathbb{C}} fg dA = 0 \tag{1.1}$$

then in a recent paper H. Hedenmalm [2] proved

Theorem 1.1. *Suppose that $f \in L^2(\mathbb{C}, e^{2\varphi})$ verifies condition (1.1) then there exists a solution to the $\bar{\partial}$ -equation $\bar{\partial}u = f$ with*

$$\int_{\mathbb{C}} |u|^2 e^{2\varphi} \Delta\varphi dA \leq \frac{1}{2} \int_{\mathbb{C}} |f|^2 e^{2\varphi} dA.$$

He adds in remark 1.3. that this theorem should generalize to the setting of several complex variables. The aim of this note is to show that he was right.

Let φ be a strictly plurisubharmonic function of class \mathcal{C}^2 in the Stein manifold Ω . Let $c_\varphi(z)$ be the smallest eigenvalue of $\partial\bar{\partial}\varphi(z)$, then $\forall z \in \Omega, c_\varphi(z) > 0$.

We denote by $L_{p,q}^{2,c}(\Omega, e^\varphi)$ the currents in $L_{p,q}^2(\Omega, e^\varphi)$ with compact support in Ω and $\mathcal{H}_p(\Omega, e^{-\varphi})$ the space of all $(p, 0), \bar{\partial}$ closed forms in $L^2(\Omega, e^{-\varphi})$.

If $p = 0, \mathcal{H}_0(\Omega) = \mathcal{H}(\Omega)$ is the space of holomorphic functions in Ω . In particular for $\omega \in L_{p,n}^2(\Omega, e^\varphi)$ we set: $\omega \perp \mathcal{H}_{n-p}(\Omega, e^{-\varphi})$ provided that $\forall h \in \mathcal{H}_{n-p}(\Omega, e^{-\varphi}), \langle \omega, h \rangle = 0$. This will be more precisely defined in the next section for Stein manifolds.

We shall prove

Theorem 1.2. *Let Ω be a pseudo convex domain in \mathbb{C}^n ; if $\omega \in L_{p,q}^{2,c}(\Omega, e^\varphi)$ with $\bar{\partial}\omega = 0$ if $q < n$ and $\omega \in L_{p,q}^2(\Omega, e^\varphi)$ with $\omega \perp \mathcal{H}_{n-p}(\Omega, e^{-\varphi})$ if $q = n$, then there is $u \in L_{p,q-1}^2(\Omega, c_\varphi e^\varphi)$ such that $\bar{\partial}u = \omega$, and*

$$\|u\|_{L^2(\Omega, c_\varphi e^\varphi)} \leq C \|\omega\|_{L^2(\Omega, e^\varphi)}.$$

Clearly Theorem 1.2 generalizes Hedenmalm’s theorem, because in one variable, we have $q = n = 1$ and no compactness assumption on the support of ω is required.

Remark 1.3. A way to see the difference between the cases $q = n$ and $q < n$ is the following.

For $q = n, \omega$ is automatically $\bar{\partial}$ -closed, but not automatically the $\bar{\partial}$ of a rapidly decaying form, which is a reason the orthogonality condition appears. On the other hand, for $q < n$ if ω is a smooth $\bar{\partial}$ -closed form with compact support, by Andreotti Grauert it is the $\bar{\partial}$ of another compactly supported form.

In fact, in the case where $\omega \in L_{p,q}^2(\Omega, e^\varphi), \bar{\partial}\omega = 0$ and $q < n$, I don’t know if there is a solution $u \in L_{p,q-1}^2(\Omega, c_\varphi e^\varphi)$ such that $\bar{\partial}u = \omega$ when ω is not compactly supported. Never the less I have the feeling that, in the case of $\Omega = \mathbb{C}^n$, this is true.

In the case Ω is a Stein manifold, the result is more restrictive: we need to take a p.s.h. exhaustion function φ in $\mathcal{C}^\infty(\Omega)$. These φ always exist in a Stein manifold by theorem 5.2.10 in [3].

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