

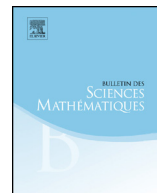


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The weighted composition operators as intertwining operators for holomorphic Lie group representations

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ABSTRACT

Let D_1 and D_2 be domains in \mathbb{C}^n and let ζ, η be holomorphic functions on D_1 such that $\eta(D_1) \subset D_2$ and $\zeta : D_1 \rightarrow \mathbb{C}$. In this paper, we determine necessary and sufficient conditions on ζ, η in order that the weighted composition operator $W_{\zeta, \eta}$ induced by ζ and η be an intertwining operator of holomorphic Lie group representations having the form $(T_g^{(j)} F)(z) = h_g^{(j)}(z) F(k_g^{(j)}(z))$, $j = 1, 2$, where $h_g^{(j)} : D_j \rightarrow \mathbb{C}$ and $k_g^{(j)} : D_j \rightarrow D_j$ are holomorphic on D_j and g is an element of the Lie group G . Furthermore, we examine conditions on ζ, η to ensure that $W_{\zeta, \eta}$ is also an intertwining operator for the infinitesimal representation of T_g given by

$$(\rho^{(j)}(v)F)(z) = \left. \frac{d}{d\epsilon} \right|_{\epsilon=0} T_{g_\epsilon}^{(j)} F(z),$$

where $(g_\epsilon)_{\epsilon \in \mathbb{R}}$ is a smooth one-parameter subgroup of G such that $v = \left. \frac{d}{d\epsilon} \right|_{\epsilon=0} g_\epsilon$ belongs to the Lie algebra of G .

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0. Introduction

Our project deals with the study of intertwining operators for the square integrable holomorphic Lie group representations as well as for their infinitesimal representations established in [2,7]. Given a Lie group G and a positive real measure μ on a domain D in the Euclidean space $\mathbb{C}^n \cong \mathbb{R}^{2n}$, we denote by $\mathcal{H}(D)$ the vector space of holomorphic functions $F : D \rightarrow \mathbb{C}$. If $|F|^2$ is μ -integrable, we denote $F \in \mathcal{H}(D) \cap L^2(\mu)$. The space $\mathcal{H}(D) \cap L^2(\mu)$ with the hermitian form

$$(F_1, F_2)_\mu = \int_D F_1(z) \overline{F_2(z)} d\mu(z) \quad \text{for } F_1, F_2 \in \mathcal{H}(D) \cap L^2(\mu), \quad (0.1)$$

is a Hilbert space. Consider operators $T_g : \mathcal{H}(D) \rightarrow \mathcal{H}(D)$ where $g \in G$ and such that

$$1. \quad (T_g F)(z) = h_g(z) F(k_g(z)), \quad \forall F \in \mathcal{H}(D) \quad (0.2)$$

where $h_g : D \rightarrow \mathbb{C}$ and $k_g : D \rightarrow D$ are holomorphic and k_g is an automorphism of D .

2. For $g_1, g_2 \in G$, we have $T_{g_1}(T_{g_2} F) = T_{g_1 g_2} F$.

3. T_g is unitary, if $F \in \mathcal{H}(D) \cap L^2(\mu)$

$$\int_D |(T_g F)(z)|^2 d\mu(z) = \int_D |F(z)|^2 d\mu(z), \quad \forall g \in G. \quad (0.3)$$

Let e denote the neutral element of G then $(T_e F)(z) = F(z)$. Therefore, T_g is invertible with $(T_g)^{-1} = T_{g^{-1}}$. The conditions (1) and (2) together imply

$$h_{g_1}(z) h_{g_2}(k_{g_1}(z)) = h_{g_1 g_2}(z) \quad \text{and} \quad k_{g_2}(k_{g_1}(z)) = k_{g_1 g_2}(z). \quad (0.4)$$

When (1), (2), (3) are satisfied, we say that μ is *unitarising* for the representation T_g , and we denote the unitary representation of G by (T_g, μ) (see [2,3,7]). This is the case of (0.5) below. Let $k_g(z) = g^{-1}.z$, where $g.z$ is a left action (respectively $k_g(z) = z.g$ a right action) of G on the domain D and let $k'_g(z)$ be the complex Jacobian of $k_g(z)$. For $\varphi, F \in \mathcal{H}(D)$ and $\alpha \in \mathbb{R}$, we put

$$(T_g^{\alpha, \varphi} F)(z) = (\det(k'_g(z)))^\alpha e^{\varphi(k_g(z)) - \varphi(z)} F(k_g(z)). \quad (0.5)$$

We have shown in [2] that if there exists a holomorphic map $\Lambda : D \times D \rightarrow \mathbb{C}$ such that $z \rightarrow \Lambda(z, \bar{z})$ on D satisfies

$$|\det(k'_g(z))|^{2\alpha-2} |\Lambda(z, \bar{z})| = |\Lambda(k_g(z), \overline{k_g(z)})|, \quad \forall g \in G, \quad \forall z \in D, \quad (0.6)$$

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