

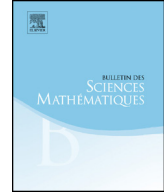


ELSEVIER

Contents lists available at ScienceDirect

Bulletin des Sciences Mathématiques

www.elsevier.com/locate/bulsci



Maximal multiplier operators in $L^{p(\cdot)}(\mathbb{R}^n)$ spaces [☆]



Amiran Gogatishvili ^{a,*}, Tengiz Kopaliani ^b

^a *Institute of Mathematics of the Academy of Sciences of the Czech Republic, Žitná 25, 115 67 Prague 1, Czech Republic*

^b *Faculty of Exact and Natural Sciences, I. Javakishvili Tbilisi State University, University St. 2, 0143 Tbilisi, Georgia*

ARTICLE INFO

Article history:

Received 28 December 2013
Available online 17 April 2015

MSC:
42B25
46E30

Keywords:

Spherical maximal function
Variable Lebesgue spaces
Boundedness result

ABSTRACT

In this paper we study some estimates of norms in variable exponent Lebesgue spaces for maximal multiplier operators. We will consider the case when multiplier is the Fourier transform of a compactly supported Borel measure.

© 2015 Elsevier Masson SAS. All rights reserved.

1. Introduction

Let $f^\wedge(\xi) = \int_{\mathbb{R}^n} e^{-2\pi i x \cdot \xi} f(x) dx$ be a Fourier transform of f . Given a multiplier $m \in L^\infty(\mathbb{R}^n)$, we define the operators M_t , $t > 0$, by $(M_t f)^\wedge(\xi) = \widehat{f}(\xi)m(t\xi)$ and the maximal multiplier operator

[☆] The research was in part supported by the grants No. 13/06 and No. 31/48 of the Shota Rustaveli National Science Foundation. The research of A. Gogatishvili was partially supported by the grant P201/13/14743S of the Grant Agency of the Czech Republic and RVO: 67985840.

* Corresponding author.

E-mail addresses: gogatish@math.cas.cz (A. Gogatishvili), tengiz.kopaliani@tsu.ge (T. Kopaliani).

$$\mathcal{M}_m f(x) := \sup_{t>0} |(M_t f)(x)|$$

which is well defined a priori for a Schwartz functions $S(\mathbb{R}^n)$.

It is well known, if multiplier m satisfies the well known Mihlin–Hörmander condition

$$|\partial^\alpha m(\xi)| \leq C_\alpha |\xi|^\alpha$$

for all (or sufficiently large) multiindices α , if $\mathcal{F}^{-1} f(\xi) = \int_{\mathbb{R}^n} e^{2\pi i x \cdot \xi} f(x) dx$ is a reverse Fourier transform, then the multiplier operator $f \mapsto \mathcal{F}^{-1}[m\widehat{f}]$ is bounded in $L^p(\mathbb{R}^n)$ when $1 < p < \infty$ (see [12,15,7,11]). Note that maximal operator \mathcal{M}_m formed by multiplier m with the Mihlin–Hörmander condition in general not bounded on $L^p(\mathbb{R}^n)$. The corresponding example can be found in [4].

We will consider the case when multiplier m is the Fourier transform of a compactly supported Borel measure. In this case the operator M_t , $t > 0$, we can represent as a convolution operator

$$M_t f(x) = \int_S f(x - ty) d\sigma(y),$$

where σ is a compactly supported Borel measure on the set $S \subset \mathbb{R}^n$ and $\widehat{\sigma}(\xi) = m(\xi)$. Obviously we have

$$\mathcal{M}_m f(x) \equiv \mathcal{M}_S f(x) := \sup_{t>0} \left| \int_S f(x - ty) d\sigma(y) \right|.$$

We say that σ is locally uniformly β -dimensional ($\beta > 0$) if $\sigma(B(x, R)) \leq C_\beta R^\beta$, where $B(x, R)$ is a ball of radius $R \leq 1$ centered at x . It is easy to see that a locally uniformly β -dimensional measure must be absolutely continuous with respect to β -dimensional Hausdorff measure μ_β , but such a measure need not exhibit any actual “fractal” behavior. Thus, for example, Lebesgue measure is locally uniformly β -dimensional for any $\beta < n$. We can allow $\beta = 0$ in these definitions, in which case a measure σ is uniformly 0-dimensional if and only if it is finite, and locally uniformly bounded, i.e. $\sigma(B(x, 1))$ is uniformly bounded in x .

Rubio de Francia [16] proved the following

Theorem 1.1. *If $m(\xi)$ is the Fourier transform of a compactly supported Borel measure and satisfies $|m(\xi)| \leq (1 + |\xi|)^{-a}$ for some $a > 1/2$ and all $\xi \in \mathbb{R}^n$, then the maximal operator \mathcal{M}_m maps $L^p(\mathbb{R}^n)$ to itself when $p > \frac{2a+1}{2a}$.*

The case when σ is normalized surface measure on the $(n - 1)$ -dimensional unit sphere was investigated by Stein [17]. According to Stein’s theorem for corresponding maximal operator (spherical maximal operator)

Download English Version:

<https://daneshyari.com/en/article/4668712>

Download Persian Version:

<https://daneshyari.com/article/4668712>

[Daneshyari.com](https://daneshyari.com)