

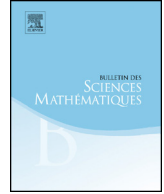


ELSEVIER

Contents lists available at ScienceDirect

Bulletin des Sciences Mathématiques

[www.elsevier.com/locate/bulsci](http://www.elsevier.com/locate/bulsci)



# Formulas for phase recovering from phaseless scattering data at fixed frequency



R.G. Novikov<sup>a,b,\*</sup>

<sup>a</sup> CNRS (UMR 7641), Centre de Mathématiques Appliquées, Ecole Polytechnique, 91128 Palaiseau, France

<sup>b</sup> IEPT RAS, 117997 Moscow, Russia

## ARTICLE INFO

### Article history:

Received 7 February 2015

Available online 17 April 2015

### MSC:

35J10

35P25

35R30

81U40

### Keywords:

Schrödinger equation

Monochromatic scattering data

Phase recovering

Phaseless inverse scattering

## ABSTRACT

We consider quantum and acoustic wave propagation at fixed frequency for compactly supported scatterers in dimension  $d \geq 2$ . In these framework we give explicit formulas for phase recovering from appropriate phaseless scattering data. As a corollary, we give global uniqueness results for quantum and acoustic inverse scattering at fixed frequency without phase information.

© 2015 Elsevier Masson SAS. All rights reserved.

## 1. Introduction

We consider the equation

$$-\Delta\psi + v(x)\psi = E\psi, \quad x \in \mathbb{R}^d, \quad d \geq 2, \quad E > 0, \quad (1.1)$$

\* Correspondence to: CNRS (UMR 7641), Centre de Mathématiques Appliquées, Ecole Polytechnique, 91128 Palaiseau, France.

E-mail address: [novikov@cmap.polytechnique.fr](mailto:novikov@cmap.polytechnique.fr).

where  $\Delta$  is the Laplacian,  $v$  is a scalar potential such that

$$v \in L^\infty(\mathbb{R}^d), \quad \text{supp } v \subset D, \\ D \text{ is an open bounded domain in } \mathbb{R}^d. \tag{1.2}$$

Eq. (1.1) can be considered as the quantum mechanical Schrödinger equation at fixed energy  $E$ .

Eq. (1.1) can also be considered as the acoustic equation at fixed frequency  $\omega$ . In this setting

$$E = \left(\frac{\omega}{c_0}\right)^2, \quad v(x) = (1 - n^2(x))\left(\frac{\omega}{c_0}\right)^2, \tag{1.3}$$

where  $c_0$  is a reference sound speed,  $n(x)$  is a scalar index of refraction.

For Eq. (1.1) we consider the classical scattering solutions  $\psi^+$  continuous and bounded on  $\mathbb{R}^d$  and specified by the following asymptotics as  $|x| \rightarrow \infty$ :

$$\psi^+(x, k) = e^{ikx} + c(d, |k|) \frac{e^{i|k||x|}}{|x|^{(d-1)/2}} f(k, |k| \frac{x}{|x|}) + O\left(\frac{1}{|x|^{(d+1)/2}}\right), \\ x \in \mathbb{R}^d, \quad k \in \mathbb{R}^d, \quad k^2 = E, \quad c(d, |k|) = -\pi i (-2\pi i)^{(d-1)/2} |k|^{(d-3)/2}, \tag{1.4}$$

where a priori unknown function  $f = f(k, l)$ ,  $k, l \in \mathbb{R}^d$ ,  $k^2 = l^2 = E$ , arising in (1.4) is the classical scattering amplitude for (1.1).

In order to find  $\psi^+$  and  $f$  from  $v$  one can use the following Lippmann–Schwinger integral equation (1.5) and formula (1.7) (see, e.g., [4,10]):

$$\psi^+(x, k) = e^{ikx} + \int_D G^+(x - y, k) v(y) \psi^+(y, k) dy, \tag{1.5}$$

$$G^+(x, k) \stackrel{\text{def}}{=} -(2\pi)^{-d} \int_{\mathbb{R}^d} \frac{e^{i\xi x} d\xi}{\xi^2 - k^2 - i0} = G_0^+(|x|, |k|), \tag{1.6}$$

where  $x \in \mathbb{R}^d$ ,  $k \in \mathbb{R}^d$ ,  $k^2 = E$ , and  $G_0^+$  depends also on  $d$ ;

$$f(k, l) = (2\pi)^{-d} \int_D e^{-il y} v(y) \psi^+(y, k) dy, \tag{1.7}$$

where  $k \in \mathbb{R}^d$ ,  $l \in \mathbb{R}^d$ ,  $k^2 = l^2 = E$ .

We recall that  $\psi^+$  describes scattering of the incident plane waves  $e^{ikx}$  on the potential  $v$ . And the second term of the right-hand side of (1.4) describes the scattered spherical waves.

In addition to  $\psi^+$ , we consider also the function  $R^+$  describing scattering of spherical waves generated by point sources. The function  $R^+ = R^+(x, x', E)$ ,  $x \in \mathbb{R}^d$ ,  $x' \in \mathbb{R}^d$ , can be defined as the Schwartz kernel of the standard resolvent  $(-\Delta + v - E - i0)^{-1}$ . Note

Download English Version:

<https://daneshyari.com/en/article/4668737>

Download Persian Version:

<https://daneshyari.com/article/4668737>

[Daneshyari.com](https://daneshyari.com)