Bull. Sci. math. 140 (2016) 82–91

Contents lists available at ScienceDirect

Bulletin des Sciences Mathématiques

www.elsevier.com/locate/bulsci

On the robustness of nonuniform exponential trichotomies ☆

Luis Barreira *, Claudia Valls

Departamento de Matemática, Instituto Superior Técnico, Universidade de Lisboa, 1049-001 Lisboa, Portugal

ARTICLE INFO

Article history:
Received 22 April 2011
Available online 14 January 2015

MSC:
34D09
34D10
37D99

Keywords:
Linear perturbations
Nonuniform exponential trichotomies
Robustness

ABSTRACT

For linear equations in a Banach space, we show that the existence of a nonuniform exponential trichotomy for $x' = A(t)x$ persists under sufficiently small C^1 perturbations $B(t,\lambda)x$, in such a way that the stable, unstable and center subspaces are of class C^1 in λ.

1. Introduction

We consider linear equations

$$x' = \big[A(t) + B(t,\lambda)\big]x \tag{1}$$

in a Banach space, obtained from a C^1 perturbation of the equation

$$x' = A(t)x. \tag{2}$$

☆ Partially supported by FCT (grant PTDC/MAT/117106/2010) and by CAMGSD.

* Corresponding author.
E-mail addresses: barreira@math.ist.utl.pt (L. Barreira), cvalls@math.ist.utl.pt (C. Valls).

http://dx.doi.org/10.1016/j.bulsci.2015.01.001

Here λ is a parameter. Our aim is to show that the existence of a nonuniform exponential trichotomy for Eq. (2) persists under sufficiently small C^1 perturbations $B(t, \lambda)x$ with an exponential decay in time, in such a way that the stable, unstable and center subspaces associated to the nonuniform exponential dichotomies in Eq. (1) are of class C^1 in λ. The proof uses former work of ours in [4] concerning the robustness of exponential dichotomies.

Due to the central role played by the notion of exponential trichotomy in the theory of center manifolds and its applications (we refer the reader to [5] for details and references), it is important to understand how exponential trichotomies vary under perturbations, and also whether center manifolds persist. Indeed, center manifold theorems are powerful tools in the analysis of the asymptotic behavior of a dynamical system. Namely, when a linear equation has no unstable directions, all solutions of any sufficiently small perturbation converge to the center manifold, and thus the stability is completely determined by the behavior on any center manifold.

We note that the study of robustness has a long history. In particular, the problem was discussed by Massera and Schäffer [9] (building on earlier work of Perron [12]; see also [10]), Coppel [7], and in the case of Banach spaces by Dalec'kiĭ and Kreĭn [8], with different approaches and successive generalizations. For more recent works we refer to [2,6,11,13,14] and the references therein. With the exception of [2], all these works consider only the case of uniform exponential behavior. We refer the reader to [1,3] for related discussions on the ubiquity of the nonuniform exponential behavior, particularly in the context of ergodic theory.

2. Basic notions and former results

Let $\mathcal{B}(X)$ be the space of bounded linear operators in the Banach space X. We consider the linear equation (2), where $A\colon \mathbb{R} \to \mathcal{B}(X)$ is a continuous function. We note that each solution of (2) is defined on the whole $\mathbb{R}$. We denote by $T(t,s)$ the associated evolution operator, that is, the linear operator such that $T(t,s)x(s) = x(t)$ for every $t, s \in \mathbb{R}$, where $x(t)$ is any solution of Eq. (2). Clearly, $T(t,t) = \mathrm{Id}$, and

$$T(t,\tau)T(\tau,s) = T(t,s), \quad t,\tau,s \in \mathbb{R}.$$

We start by recalling a result in [4] on the robustness of nonuniform exponential dichotomies. We first recall that Eq. (2) is said to admit a *nonuniform exponential dichotomy* if there exist projections $\bar{P}(t)\colon X \to X$ varying continuously with $t \in \mathbb{R}$ such that

$$T(t,s)\bar{P}(s) = \bar{P}(t)T(t,s)$$

for every $t, s \in \mathbb{R}$, and there exist constants $\alpha, C > 0$ and $\beta \geq 0$ such that for every $t, s \in \mathbb{R}$ with $t \geq s$ we have

$$\left\|T(t,s)\bar{P}(s)\right\| \leq Ce^{-\alpha(t-s)+\beta|s|},$$