Bull. Sci. math. 140 (2016) 99–117

Contents lists available at ScienceDirect

Bulletin des Sciences Mathématiques

www.elsevier.com/locate/bulsci

Periodic solutions for discontinuous perturbations of the relativistic operator

Petru Jebelean [a,b,*], Jean Mawhin [c], Călin Şerban [a]

[a] *Department of Mathematics, West University of Timişoara, 4, Blvd. V. Pârvan, 300223-Timişoara, Romania*
[b] *Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Romania*
[c] *Département de Mathématique, Université Catholique de Louvain, 2, Chemin du Cyclotron, B-1348 Louvain-la-Neuve, Belgique*

A R T I C L E   I N F O

Article history:
Received 14 November 2014
Available online 11 February 2015

MSC:
34A60
47H14
49J52
49K21

Keywords:
Relativistic operator
Periodic solutions
Non-smooth critical point theory
Differential inclusions
Variational method
Filippov type solutions

A B S T R A C T

In this paper we study the existence and multiplicity of periodic solutions for discontinuous perturbations of the operator $u \mapsto \left(\frac{u'}{\sqrt{1-|u'|^2}}\right)'$. The results are obtained by reduction to an equivalent non-singular problem and using the non-smooth critical point theory. Some illustrative examples concerning Filippov type solutions are also provided.

© 2015 Elsevier Masson SAS. All rights reserved.

* Corresponding author.
E-mail addresses: jebelean@math.uvt.ro (P. Jebelean), jean.mawhin@uclouvain.be (J. Mawhin), cserban2005@yahoo.com (C. Şerban).

http://dx.doi.org/10.1016/j.bulsci.2015.02.002
0007-4497/© 2015 Elsevier Masson SAS. All rights reserved.

1. Introduction

In this paper we deal with differential inclusions systems of type

$$-(\varphi(u'))' \in \partial F(t,u), \qquad u(0)-u(T)=0=u'(0)-u'(T), \tag{1.1}$$

where φ is defined by

$$\varphi(y)=\frac{y}{\sqrt{1-|y|^2}} \qquad (y\in B(1)) \tag{1.2}$$

and the mapping $F:[0,T]\times\mathbb{R}^N\to\mathbb{R}$ is assumed to satisfy the hypothesis:

(H_F) *(i)* $F(\cdot,x):[0,T]\to\mathbb{R}$ *is measurable for every* $x\in\mathbb{R}^N$ *and* $F(\cdot,0)=0$;
(ii) there exists $\alpha\in L^1([0,T];\mathbb{R})$ *such that for all* $t\in[0,T]$ *and* $x,y\in\mathbb{R}^N$*, it holds*

$$|F(t,x)-F(t,y)|\le\alpha(t)|x-y|.$$

Here and below, $|\cdot|$ stands for the Euclidean norm on $\mathbb{R}^N$, $B(\sigma)\subset\mathbb{R}^N$ denotes the open ball of center 0 and radius σ and $\partial F(t,x)$ stands for the generalized Clarke gradient of $F(t,\cdot)$ at $x\in\mathbb{R}^N$.

In recent years the study of boundary value problems involving the relativistic operator

$$u\mapsto\mathcal{R}u:=(\varphi(u'))'$$

has captured a special attention. Mainly, the obtained results are concerned with the existence and multiplicity of solutions for problems involving continuous perturbations of $\mathcal{R}$ and less of them deal with discontinuous perturbations – case in which a differential inclusion problem occurs. In this second direction, we refer to paper [4], where the authors deal with the set-valued problem

$$(\varphi(u'))'\in g(t,u,u'), \qquad u(0)-u(T)=0=u'(0)-u'(T),$$

where g is a multifunction with nonempty, compact and convex-valued. Their approach relies on fixed-point techniques combined with the lower and upper solutions method. Also, we note paper [13] for other results concerning differential inclusions systems involving singular ϕ-Laplacians and based upon topological methods.

Unlike these articles, the method that we shall use in this paper is variational. The main goal is to extend the results obtained in [8] to the non-smooth problem (1.1). In this view, the idea is to reduce the singular problem (1.1) to an equivalent non-singular one to which we can apply the non-smooth critical point theory developed by Chang [2]. In the first result of the paper, the solutions which we obtain appear either as minimizers

Download English Version:

https://daneshyari.com/en/article/4668749

Download Persian Version:

https://daneshyari.com/article/4668749

Daneshyari.com