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Persistent centers of complex systems ☆

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Abstract

On the basis of some works on persistent centers and weakly persistent centers, in this paper we discuss a generalized version of persistent center and weakly persistent center for complex planar differential systems, in which conjugacy of variables may not be required. We give some complex systems which have a persistent center or weakly persistent center at the origin. Then, we find all conditions of persistent center for cubic systems and all conditions of weakly persistent center for complex cubic Lotka–Volterra system. Relations between complex systems and real ones are given concerning persistent centers and weakly persistent centers.

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1. Introduction

Consider an analytic real system of the form

$$\dot{u} = -v + P(u, v), \qquad \dot{v} = u + Q(u, v), \tag{1.1}$$

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where $u, v \in \mathbb{R}$ and P, Q are analytic functions whose series expansions start from degree greater than or equal to 2. The origin $O : (0,0)$ is called a *center* of system (1.1) if it is surrounded by a family of periodic orbits, or equivalently [15,17], if there exists an analytic first integral of the form

$$\phi(u,v) = \frac{u^2+v^2}{2} + \text{h.o.t.}$$

The center problem has been investigated intensively since the era of Lyapunov and Poincaré. However, up to now, only a few families of polynomial planar differential systems such as the quadratic system [10,11], linear center perturbed by third-degree homogeneous polynomials [16], cubic Kolmogorov systems [14], and some others (see [1,3,9,18] and references therein) are classified for centers. One of main difficulties arising in the problem of classification of centers of polynomial systems is tremendous computations which are faced in the decomposition of algebraic sets, which as rule cannot be completed even at very powerful computers with the most efficient tools of computational algebra.

Recently, the concepts of persistent center and weakly persistent center were introduced [4], while great attention was paid to families of systems parameterized by parameters because bifurcations may occur in a neighborhood of the center variety, the set of parameters in which the origin is a center, as the structure of centers is destroyed (see e.g. [8,19,18] and references therein). Introducing the complex variable $x = u + \mathrm{i}v$ we write system (1.1) in the complex form

$$\dot{x} = \mathrm{i}x + F(x,\bar{x}), \tag{1.2}$$

where $\mathrm{i} = \sqrt{-1}$. As defined in [4], the origin O is called a *persistent center* (resp. *weakly persistent center*) of system (1.1) if it is a center of the parameterized system

$$\dot{x} = \mathrm{i}x + \lambda F(x,\bar{x}) \tag{1.3}$$

for all $\lambda \in \mathbb{C}$ (resp. $\mathbb{R}$). Obviously, a persistent center implies a weakly persistent center.

It is also interesting to discuss the corresponding object in a general complex setting. Consider the general analytic complex system

$$\dot{x} = \mathrm{i}x + F(x,y), \qquad \dot{y} = -\mathrm{i}y + G(x,y), \quad x, y \in \mathbb{C}, \tag{1.4}$$

where F, G are analytic functions whose series expansions start from terms of degree at least 2. System (1.2) is a special case of (1.4), where $y = \bar{x}$, $G(x,\bar{x}) = \overline{F(x,\bar{x})}$. Without loss of generality, let

$$F(x,y) = \sum_{i+h=2}^{\infty} a_{ih} x^i y^h, \qquad G(x,y) = \sum_{i+h=2}^{\infty} b_{ih} x^i y^h, \tag{1.5}$$

where a_{ih}, b_{ih} are complex parameters. Following Dulac [7,13], we say that the origin O is a *center* of the complex system (1.4) if there exists an analytic first integral of the form

$$\psi(x,y) = \mathrm{i}xy + \text{h.o.t.} \tag{1.6}$$

Thus, the center problem for system (1.4) is equivalent to finding parameter conditions under which (1.4) has an analytic first integral of form (1.6). The problem was solved for quadratic system (1.4) in [2,7], for system (1.4) with homogeneous cubic nonlinearities F and G in [20], and for the Lotka–Volterra system with homogeneous quintic nonlinearities [9] and for few other particular polynomial families (see e.g. [3,12,18] and the references therein). Generally, as shown

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