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Weighted estimates for commutators of some singular integrals related to Schrödinger operators

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Abstract

Let $L = -\Delta + V$ be a Schrödinger operator with a non-negative potential V satisfying some appropriate reverse Hölder inequality. In this paper, we study the boundedness of the commutators of some singular integrals associated to L such as the Riesz transforms and fractional integrals with the new BMO functions introduced in Bongioanni et al. (2011) [1] on the weighted spaces $L^p(w)$ where w belongs to the new classes of weights introduced by Bongioanni et al. (2011) [2].

MSC: 42B35; 35J10

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1. Introduction

Let $L = -\Delta + V$ be the Schrödinger operators on $\mathbb{R}^n$ with $n \geqslant 3$ where the potential V is in the reverse Hölder class RH_q for some $q > n/2$, i.e., V satisfies the reverse Hölder inequality

$$\left(\frac{1}{|B|}\int_B V(y)^q\,dy\right)^{1/q} \leqslant \frac{C}{|B|}\int_B V(y)\,dy$$

for all balls $B \subset \mathbb{R}^n$.

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In this paper, we consider the following singular integrals associated to L:

(i) The Riesz transforms $R = \nabla L^{-1/2}$ and their adjoint $R^* = L^{-1/2}\nabla$.
(ii) Fractional integrals $I_\alpha f(x) = L^{-\alpha/2} f(x)$ for $0 < \alpha < n$.

In the classical case when $V = 0$, it has been shown that Riesz transforms R and their commutators R_b with BMO functions b is bounded on $L^p(w)$ for all $1 < p < \infty$ and w in the Muckenhoupt classes A_p, see for example [9]. Also, the classical fractional integrals and their commutators with BMO functions b are bounded from $L^p(w^p)$ to $L^q(w^q)$ for all $1 < p < n/\alpha$, $1/p - 1/q = \alpha/n$ and $w \in A_{1+1/p'} \cap RH_q$, or equivalently $w^q \in A_{1+\frac{q}{p'}}$, where A_p is the Muckenhoupt class of weights, see for example [10,11]. Recall that a non-negative and locally integrable function w is said to be in the Muckenhoupt A_p classes with $1 \leqslant p < \infty$, if the following inequality holds for all balls $B \subset \mathbb{R}^n$

$$\Big(\int_B w\Big)^{1/p}\Big(\int_B w^{-\frac{1}{p-1}}\Big)^{1/p'} \leqslant C|B|. \tag{1}$$

Recently, in [2], a new class of weights associated to Schrödinger operators L has been introduced. According to [2], the authors defined the new classes of weights $A_p^L = \bigcup_{\theta>0} A_p^{L,\theta}$ for $p \geqslant 1$, where $A_p^{L,\theta}$ is the set of those weights satisfying

$$\Big(\int_B w\Big)^{1/p}\Big(\int_B w^{-\frac{1}{p-1}}\Big)^{1/p'} \leqslant C|B|\Big(1 + \frac{r}{\rho(x)}\Big)^{\theta} \tag{2}$$

for all balls $B = B(x, r)$. We denote $A_\infty^L = \bigcup_{p\geqslant 1} A_p^L$ where the critical radius function $\rho(\cdot)$ is defined by

$$\rho(x) = \sup\Big\{r > 0\colon \frac{1}{r^{n-2}} \int_{B(x,r)} V \leqslant 1\Big\}, \quad x \in \mathbb{R}^n, \tag{3}$$

see [8].

It is easy to see that in certain circumstances the new class A_p^L is larger than the Muckenhoupt class A_p. The following properties hold for new classes A_p^L, see [2, Proposition 5].

Proposition 1.1. *The following statements hold*:

i) $A_p^L \subset A_q^L$ *for* $1 \leqslant p \leqslant q < \infty$.
ii) *If* $w \in A_p^L$ *with* $p > 1$ *then there exists* $\epsilon > 0$ *such that* $w \in A_{p-\epsilon}^L$. *Consequently,* $A_p^L = \bigcup_{q<p} A_q^L$.

For new classes A_p^L, the weighted norm inequalities for the some singular integrals associated to L were investigated.