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Drift parameter estimation for infinite-dimensional fractional Ornstein–Uhlenbeck process

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Abstract

We analyze the least squares estimator for the drift parameter of an infinite-dimensional fractional Ornstein–Uhlenbeck process with Hurst parameter $H \geqslant \frac{1}{2}$. This estimator can be expressed in terms of a divergence integral with respect to the fractional Brownian motion. Using some recently developed criteria based on Malliavin calculus and Wiener–Itô chaos expansion, we prove the strong consistency and the asymptotic normality of the estimator.

MSC: 60H15; 60H07; 60G35

Keywords: Fractional Brownian motion; Parameter estimation; Stochastic evolution equations; Malliavin calculus; Multiple Wiener–Itô integrals; Strong consistency; Asymptotic normality

1. Introduction

The aim of the paper is to study parameter estimation aspects for stochastic partial differential equations driven by fractional Brownian motion. While statistical inference for finite dimensional stochastic equations driven by standard Brownian motion has been widely developed,

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the scientific literature related to the parameter estimation for SPDEs (or, in general, infinite-dimensional stochastic equations) driven by Wiener process is relatively recent. Let us mention the pioneering work [17] where the minimum contrast estimator has been studied provided the strong solution exists or the papers [6,7] where least squares estimators for controlled linear stochastic evolution equations were proposed. In the paper [12] the maximum likelihood estimator (MLE) for the drift parameter is dealt with for a stochastic parabolic equation driven by a space–time white noise. We also mention the work [14] for a particular case of the heat equation. Other references are [27] for MLE and Bayes estimators based on discrete observations, [10] for a controlled semilinear equation with drift depending on the unknown parameter, [2] for diagonalizable bilinear parabolic equations or [15] for the case of a nonlinear stochastic differential equation.

Statistical inference results for equations driven by fractional Brownian motion (fBm) are obviously more recent. They appeared only after the development of stochastic calculus with respect to the fBm allowed to study such systems. For finite dimensional equation, there are several approaches to estimate the parameters of the model. Let us mention

- The MLE approach in [16], [28] or [31]. In general the techniques used to construct maximum likelihood estimators for the drift parameter are based on Girsanov transforms for fractional Brownian motion and depend on the properties of the deterministic fractional operators (determined by the Hurst parameter) related to the fBm. As usual, the MLE is not easily computable.
- A pseudo-MLE approach based on the discretization of the equation in [1,30]. This approach allows to simulate better the estimator obtained. Some numerical results are presented in [1] and [30] as well.
- Recently, a least squares approach has been proposed in [11]. The study of the asymptotic properties of the estimator is based on certain criteria formulated in terms of the Malliavin calculus (see [23]). The computation of the estimator is difficult, however, in [11] the authors show that this estimator is asymptotically close to another estimator, which in principle can be computed.
- Other type of estimators, such as minimum L^1-norm estimator, contrast estimators, etc., can be found in [29].

The references related to the infinite-dimensional stochastic equations driven by fBm are very limited. We mention the works [20,21] for estimation of the drift parameter of infinite-dimensional Ornstein–Uhlenbeck process. The estimators proposed in these papers are basically of minimum contrast type (cf. also [17] in the case $H = \frac{1}{2}$). MLE estimators for various types of equations with fractional, mostly additive, noise have been studied in [3,4].

Our work extends the one-dimensional case the approach in [11] to infinite dimensions and our main examples are stochastic linear parabolic-type equations with the parameter in the drift. More specifically, we treat the problem of estimation of a real parameter θ in the drift of an infinite-dimensional fractional Ornstein–Uhlenbeck process which is defined as the solution of the equation

$$dX(t) = \theta AX(t)\,dt + \Phi\,dB_t^H, \quad X(0) = x_0 \in V$$

and takes values in a real separable Hilbert space V. Here the process $(B_t^H)_{t\geqslant 0}$ is a fractional Brownian motion (possibly, cylindrical) on V with $H \geqslant \frac{1}{2}$ and the operators A, Φ are defined on V or a subspace of V. A minimum contrast estimator for θ has been proposed in [20,21]