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A new type of contractive multivalued operators

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Abstract

The aim of this paper is to introduce a new type of multivalued operators similar to those of Kikkawa–Suzuki type and to present some basic problems of the fixed point and strict fixed point for them. Obtained results generalize, complement and extend classical results given by Ćirić [Lj.B. Ćirić, Fixed points for generalized multi-valued contractions, Mat. Vesnik 9 (24) (1972) 265–272] or Nadler [S.B. Nadler Jr., Multi-valued contraction mappings, Pacific J. Math. 30 (1969) 475–488], as well as recent results given by Kikkawa and Suzuki [M. Kikkawa, T. Suzuki, Three fixed point theorems for generalized contractions with constants in complete metric spaces, Nonlinear Anal. 69 (2008) 2942–2949], Moţ and Petruşel [G. Moţ, A. Petruşel, Fixed point theory for a new type of contractive multivalued operators, Nonlinear Anal. 70 (2009) 3371–3377]. Applications to certain functional equations arising in dynamic programming are also considered.

Résumé

Le but de cet article est d'introduire un nouveau type d'opérateurs multivoques similaires ceux de Kikkawa–Suzuki type et de présenter certains problémes fondamentaux de la point fixe et stricte point fixe pour eux. Les résultats obtenus généraliser, compléter et étendre les résultats classiques donnés par Ćirić [Lj.B. Ćirić, Fixed points for generalized multi-valued contractions, Mat. Vesnik 9 (24) (1972) 265–272] ou Nadler [S.B. Nadler Jr., Multi-valued contraction mappings, Pacific J. Math. 30 (1969) 475–488], comme ainsi que les résultats récents donné par Kikkawa et Suzuki [M. Kikkawa, T. Suzuki, Three fixed point theorems for generalized contractions with constants in complete metric spaces, Nonlinear Anal. 69 (2008) 2942–2949], Moţ et Petruşel [G. Moţ, A. Petruşel, Fixed point theory for a new type of contractive multi-valued operators, Nonlinear Anal. 70 (2009) 3371–3377]. Applications à certaines équations fonctionnelles découlant de dynamique programmation sont également considérés.

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1. Introduction

Let (X, d) be a metric space and let $CB(X)$ (resp. $CL(X)$) denote the family of all nonempty closed bounded (resp. closed) subsets of X. For any subsets A, B of X,

$$D(A, B) := \inf\{d(a, b)\colon\ a \in A,\ b \in B\}$$

denotes the gap between the subsets A and B. In particular, if $x \in X$ then $D(x, B) := D(x, B)$. Also,

$$\rho(A, B) := \sup\{D(a, B)\colon\ a \in A\}$$

is called the generalized excess functional, and

$$H(A, B) := \max\{\rho(A, B), \rho(B, A)\}$$

is the (generalized) Pompeiu–Hausdorff functional.

It is well-known that if (X, d) is a complete metric space, then the pair $(CB(X), H)$ is a complete metric space, while $(CL(X), H)$ is a complete generalized metric space (in the sense of Luxemburg–Jung), see for example [5,9,20].

Let X, Y be two nonempty sets and $T : X \to P(Y)$. Denote by $G(T) := \{(x, y)\colon\ x \in X, y \in Tx\}$ the graph of the multivalued operator T. A selection for T is a single operator $t : X \to Y$ such that $tx \in Tx$ for each $x \in X$.

Definition 1.1. Let X be a nonempty set. If $T : X \to P(X)$ is a multivalued operator, then an element $x \in X$ is called a *fixed point* (*strict fixed point*) for T if $x \in Tx$ ($\{x\} = Tx$). We denote by $Fix(T) := \{x \in X\colon\ x \in Tx\}$ the fixed point set of T and by $SFix(T) := \{x \in X\colon\ \{x\} = Tx\}$ the set of all strict fixed points of T.

Definition 1.2. (See Rus–Petruşel–Sîntămărian [26].) Let (X, d) be a metric space and $T : X \to CL(X)$ a multivalued operator. T is called a *multivalued weakly Picard operator* (briefly MWP operator) if for all $x \in X$ and all $y \in Tx$, there exists a sequence $\{x_n\}_{n\geqslant 0}$ such that:

(i) $x_0 = x, x_1 = y$,
(ii) $x_{n+1} \in Tx_n$, for all $n \geqslant 0$,
(iii) the sequence $\{x_n\}_{n\geqslant 0}$ is convergent and its limit is a fixed point of T.

A sequence $\{x_n\}_{n\geqslant 0}$ satisfying (i) and (ii) is also called a *sequence of successive approximations* (briefly s.s.a.) of T starting from x_0.

In [26] the theory of MWP operators was presented.

In 2008 Suzuki [29] introduced a new type of mappings which generalize the well-known Banach contraction principle [1].

Theorem 1.3. *Let (X, d) be a complete metric space and $S : X \to X$. Define a nonincreasing function θ from $[0, 1)$ onto $(\frac{1}{2}, 1]$ by*