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Attractors of impulsive dissipative semidynamical systems

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Abstract

In this paper, we consider a class of dissipative semidynamical systems with impulses. First, we study the connectedness of Levinson's center of a compact dissipative system with impulses. Second, we define some types of attractors for dissipative systems and we study results which relate attractors and dissipative systems (point, bounded and compact). Finally, we apply our results for a general impulsive autonomous system and for a nonlinear reaction–diffusion equation of type $u' - \Delta u + g(u) = f$ with impulse condition.

Keywords: Semidynamical systems; Dissipative systems; Attractors; Impulses

1. Introduction

Impulsive differential equations are important to model real-world problems in science and technology. This theory has been intensively investigated. The reader may find some applications in [1,7,12,13], for instance.

The theory of impulsive differential equations is applied in the theory of dynamical systems. The action of impulses on dynamical systems has been intensively investigated, see, for instance, [2–6,9–15] and the references therein.

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In this paper we continue to study the theory of impulsive dissipative semidynamical systems. In the next lines, we describe the organization of the paper and the main results.

In the first part of this paper, we present the basis of the theory of impulsive semidynamical systems. We divide Section 2 into three parts. In Section 2.1, we give some basic definitions and notations about impulsive semidynamical systems. In Section 2.2, we discuss the continuity of a function which describes the times of meeting impulsive sets. In Section 2.3, we give some additional useful definitions and results about dissipative systems.

The second part of the paper, namely Section 3, concerns the main results. In Section 3.1 we consider a compact k-dissipative semidynamical system with impulses $(X, \pi; M, I)$ and we show that its center of Levinson is connected provided X is connected and M satisfies a special condition, see Theorem 3.4. By imposing an impulse condition on each component of X we prove the reciprocal of this result, that is, if every component C of X has the property $I(C \cap M) \subset C$, then X is connected provided J is connected, see Theorem 3.5.

In Section 3.2, we define the following concepts for impulsive systems: global attractor, Ladyzhenskaya's condition, asymptotically compact systems, completely continuous systems, weakly b-dissipative and weakly k-dissipative systems. We show that an impulsive semidynamical system is compact k-dissipative if and only if this system admits a global attractor, see Theorem 3.7. Also, we prove that an impulsive semidynamical system $(X, \pi; M, I)$ is bounded k-dissipative if and only if it is point k-dissipative, $\widetilde{D}^+(\Omega) \cap M = \emptyset$ and it satisfies the condition of Ladyzhenskaya, see Theorem 3.8. For weakly dissipative systems, we show that a system $(X, \pi; M, I)$ weakly b-dissipative and completely continuous is weakly k-dissipative, see Theorem 3.10. Theorem 3.11 shows that if a weak k-attractor of a completely continuous and weakly k-dissipative impulsive system is uniform $\widetilde{\pi}$-attracting and it does not intercept the impulsive set M, then the impulsive system admits a compact global attractor. Theorem 3.12 gives a result that relates several types of dissipative systems.

In Section 3.3, we consider an impulsive dynamical system given by

$$\begin{cases} x' = -f(x), \\ I: M \to \mathbb{R}^n, \end{cases} \tag{1.1}$$

where $f \in C^1(\mathbb{R}^n, \mathbb{R}^n)$. By supposing that system (1.1) satisfies some conditions, we prove that this system is local k-dissipative. As a corollary, we show that system (1.1) is compact k-dissipative, satisfies the condition of Ladyzhenskaya, is completely continuous, is weakly b-dissipative, is weakly k-dissipative and admits a compact global attractor, see Corollary 3.4.

Finally, in Section 3.4, we consider a nonlinear impulsive reaction–diffusion equation given by

$$\begin{cases} u' - \Delta u + g(u) = f, \\ u|_{\partial\Omega} = 0, \\ I: M \to L^2(\Omega), \end{cases} \tag{1.2}$$

where $g \in C^1(\mathbb{R}, \mathbb{R})$, $f \in L^2(\Omega)$, Ω is a bounded smooth domain of $\mathbb{R}^n$ with smooth boundary, Δ is the Laplace operator in Ω, $M \subset L^2(\Omega)$ is an impulsive set and I is the impulse function. By assuming additional conditions, we show that system (1.2) is local dissipative. If we consider $L^2(\Omega)$ with the weak topology $\sigma(L^2(\Omega), L^2(\Omega))$, we show that system (1.2) is bounded k-dissipative, see Corollary 3.6. The last result shows that system (1.2), under some hypotheses, is compact k-dissipative, satisfies the condition of Ladyzhenskaya, is completely continuous, is weakly b-dissipative, is weakly k-dissipative and admits a compact global attractor, see Corollary 3.7.