Available online at www.sciencedirect.com

ScienceDirect

Bull. Sci. math. 137 (2013) 1007–1017

BULLETIN DES SCIENCES MATHÉMATIQUES

www.elsevier.com/locate/bulsci

Existence of doubly periodic solutions for a class of telegraph system with indefinite weight

Fanglei Wang [a,*], Yukun An [b]

[a] *College of Science, Hohai University, Nanjing, 210098, PR China*

[b] *Department of Mathematics, Nanjing University of Aeronautics and Astronautics, Nanjing, 210016, PR China*

Received 21 January 2013

Available online 7 February 2013

Abstract

Using the method of monotone iteration and Schauder fixed point theorem, we establish the existence of doubly periodic solutions for a coupled nonlinear telegraph system with indefinite weight.

MSC: 34B15; 34B18

Keywords: Telegraph system; Doubly periodic solution; Cone; Fixed point theorem

1. Introduction

In this paper we are concerned with the existence of double periodic solutions for the nonlinear telegraph system

$$\begin{cases} u_{tt} - u_{xx} + c_1 u_t + a_{11}(t,x)u - a_{12}(t,x)v = \lambda a(t,x) f(v), \\ v_{tt} - v_{xx} + c_2 v_t - a_{21}(t,x)u + a_{22}(t,x)v = \tau b(t,x) g(u), \end{cases} \tag{1}$$

with doubly periodic boundary conditions

$$\begin{cases} u(t+2\pi,x) = u(t,x+2\pi) = u(t,x), \\ v(t+2\pi,x) = v(t,x+2\pi) = v(t,x), \end{cases}$$

* Corresponding author.
E-mail addresses: wang-fanglei@hotmail.com (F. Wang), anykna@nuaa.edu.cn (Y. An).

0007-4497/$ – see front matter
http://dx.doi.org/10.1016/j.bulsci.2013.02.005

where $c_1, c_2 > 0$ are constants, λ, τ are parameters, the functions $a_{11}(t,x)$, $a_{22}(t,x) \in C(R,R)$ are nonnegative, $a_{12}(t,x)$, $a_{21}(t,x)$, $a(t,x)$, $b(t,x) \in C(R^2,R)$ may change sign, $f(v)$, $g(u) \in C(R,R)$.

On the basis of the maximum principle for periodic solutions of the telegraph equations:

$$\begin{cases} u_{tt} - u_{xx} + cu_t + \dfrac{c^2}{4}u = h(t,x), \\ u(t+2\pi,x) = u(t,x+2\pi) = u(t,x), \end{cases}$$

build by R. Ortega, A.M. Robles-Perez in [1], on one hand, some authors have studied the existence and multiplicity of doubly periodic solutions for the scalar telegraph equation. For examples, using the fixed point theorem in a cone, Y. Li [2] has studied the existence of positive doubly periodic solutions of a nonlinear telegraph equations:

$$\begin{cases} u_{tt} - u_{xx} + cu_t + a(t,x)u = b(t,x)f(t,x,u), \\ u(t+2\pi,x) = u(t,x+2\pi) = u(t,x), \end{cases}$$

where $c > 0$ is a constant, $a(t,x)$, $b(t,x)$ are positive, and f satisfies the superlinear or sublinear growth conditions. In [3,4], C. Bereanu has applied the method of upper and lower solutions and Leary–Schauder degree theory to establish the existence, nonexistence and multiplicity results for the following equations:

$$\begin{cases} u_{tt} - u_{xx} + cu_t + f(t,x,u) = h(t,x) + s, \\ u(t+2\pi,x) = u(t,x+2\pi) = u(t,x), \end{cases}$$

where s is a parameter. On the other hand, F. Wang, Y. An have studied a coupled nonlinear telegraph system:

$$\begin{cases} u_{tt} - u_{xx} + c_1u_t + a_{11}(t,x)u + a_{12}(t,x)v = \lambda b_1(t,x)f(t,x,u,v), \\ v_{tt} - v_{xx} + c_2v_t + a_{21}(t,x)u + a_{22}(t,x)v = \mu b_2(t,x)g(t,x,u,v), \end{cases}$$

with doubly periodic boundary conditions

$$\begin{cases} u(t+2\pi,x) = u(t,x+2\pi) = u(t,x), \\ v(t+2\pi,x) = v(t,x+2\pi) = v(t,x), \end{cases}$$

using the fixed point theorem, the method of upper and lower solutions, Leary–Schauder degree theory, see [5–8]. In these references, the functions $a_{11}(t,x)$, $a_{22}(t,x)$, $b_1(t,x)$, $b_2(t,x)$ are positive, and $a_{12}(t,x)$, $a_{21}(t,x)$ are negative.

In this article, we will release the sign of $a_{12}(t,x)$, $a_{21}(t,x)$, $a(t,x)$ and $b(t,x)$, and prove the existence of nonnegative solutions to the telegraph system (1) using the method of monotone iteration and Schauder fixed point theorem. The idea is coming from [9,10] in which the authors studied the following elliptic system:

$$\begin{cases} -\Delta u = \lambda a(t,x)f(v), & \text{in } \Omega, \\ -\Delta v = \lambda b(t,x)g(u), & \text{in } \Omega, \\ u = v = 0, & \text{on } \partial\Omega. \end{cases}$$

This model has also been studied by many authors, see [11–13].

The paper is organized as follows: In Section 2, some preliminaries are given; in Section 3, we will give the main result and proof.