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Dimension vectors in regular components over wild Kronecker quivers

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Abstract

Let $\mathcal{K}_n$ be the so-called wild Kronecker quiver, i.e., a quiver with one source and one sink and $n \geqslant 3$ arrows from the source to the sink. The following problems will be studied for an arbitrary regular component $\mathcal{C}$ of the Auslander–Reiten quiver: (1) What is the relationship between dimension vectors and quasi-lengths of the indecomposable regular representations in $\mathcal{C}$? (2) For a given natural number d, is there an upper bound of the number of indecomposable representations in $\mathcal{C}$ with the same length d? (3) When do the sets of the dimension vectors of indecomposable representations in different regular components coincide?

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1. Introduction

Quivers (without oriented cycles) and their representations over a field play an important role in representation theory of finite-dimensional algebras; they also occur in other domains including Kac–Moody Lie algebras, quantum groups. Gabriel characterized quivers of finite representation type [7], that is, having only finitely many isomorphism classes of indecomposable representations: such quivers are exactly disjoint unions of Dynkin diagrams of types A_n, D_n, E_6, E_7, E_8, equipped with arbitrary orientations. Moreover, the isomorphism classes of

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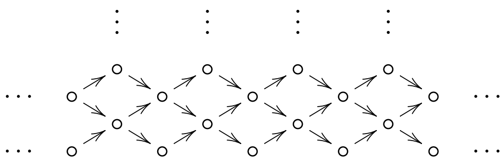
indecomposable representations correspond bijectively to the positive roots of the associated root systems. Whereas, the underlying graphs of quivers of tame representation type were shown [3,8] to be disjoint unions of Euclidean diagrams of types $\widetilde{A}_n$, $\widetilde{D}_n$, $\widetilde{E}_6$, $\widetilde{E}_7$, $\widetilde{E}_8$. Any quiver, which is neither of finite nor of tame type, is of wild representation type. Kac generalized Gabriel's result to arbitrary quivers (exposed in [5]): the dimension vectors of indecomposable representations are positive roots, and moreover, for each positive real root there is a unique isomorphism class of indecomposable representation, whereas for each positive imaginary root there are infinitely many ones.

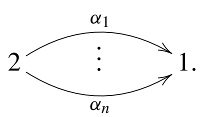
Representations of quivers of finite and tame types are well understood. However, almost all quivers are of wild representation type and their representations are very complicated. The so-called Auslander–Reiten (*AR*) quiver of a connected wild quiver consists a preprojective component containing all indecomposable projective representations, a preinjective component containing all indecomposable injective representations, and infinitely many regular components of the form $\mathbb{Z}\mathbb{A}_\infty$:



Kac's theorem tells the existence of indecomposable representations for every positive root. But it doesn't help to determine the positions of the indecomposable representations in the components (more interesting, regular components) of the AR-quiver.

Given a wild quiver Q, a regular component $\mathcal{C}$ of its AR-quiver and a natural number $d > 0$, let $\beta_{Q,\mathcal{C}}(d)$ denote the number of the indecomposable representations in $\mathcal{C}$ with length d, and let $\beta_{Q,d} = \sup_{\mathcal{C}} \beta_{Q,\mathcal{C}}(d)$ and $\beta_Q = \sup_d \beta_{Q,d}$, where the supremes are taken over all regular components and all natural numbers d, respectively. In [10], it was proved (for any Artin wild hereditary algebra) that the indecomposable representations in a regular component of the AR-quiver are uniquely determined by their dimension vectors. It follows immediately that $\beta_{Q,\mathcal{C}}(d)$ is finite for any regular component $\mathcal{C}$ and $d > 0$ and $\beta_{Q,d}$ is obviously finite since Q has only finitely many vertices. However, it is not known yet if β_Q is finite.

In this paper, we will focus on a class of quivers and their representations over an algebraically closed field k: the so-called Kronecker quivers $\mathcal{K}_n$ with n arrows



A finite-dimensional representation of $\mathcal{K}_n$ over k is simply called a $\mathcal{K}_n$-module. It is well known that $\mathcal{K}_n$ is of wild (resp. tame, finite) representation type if $n \geqslant 3$ (resp. $n = 2$, $n = 1$). From now on, unless stated otherwise, we always consider wild Kronecker quivers, i.e., $n \geqslant 3$.

One of the main aims of the paper is to study the relationship between the dimension vectors of indecomposable $\mathcal{K}_n$-modules and their quasi-lengths, that is, on which layers of the regular components they locate. For each $r \geqslant 0$, let A_r be the r-th number appearing in the ordered set of the entries of the dimension vectors of the indecomposable preprojective $\mathcal{K}_n$-modules (see Section 2). We will see that if (a, b) is an imaginary root, then there is an indecomposable module M with dimension vector (a, b) and quasi-length r if and only if A_r is a common divisor of a

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