C. R. Acad. Sci. Paris, Ser. I 353 (2015) 551–555

Contents lists available at ScienceDirect

C. R. Acad. Sci. Paris, Ser. I

www.sciencedirect.com

Dynamical systems/Probability theory

Equivalence of Palm measures for determinantal point processes associated with Hilbert spaces of holomorphic functions

Équivalence de measures de Palm pour les processus déterminantaux associés aux espaces de Hilbert des fonctions holomorphes

Alexander I. Bufetov [a,b,c,d,e], Yanqi Qiu [a]

[a] *Aix-Marseille Université, Centrale Marseille, CNRS, I2M, UMR7373, 39, rue Frédéric-Joliot-Curie, 13453 Marseille, France*
[b] *Steklov Institute of Mathematics, Moscow, Russian Federation*
[c] *Institute for Information Transmission Problems, Moscow, Russian Federation*
[d] *National Research University Higher School of Economics, Moscow, Russian Federation*
[e] *Rice University, Houston, TX, United States*

A R T I C L E I N F O

Article history:
Received 29 January 2015
Accepted after revision 4 March 2015
Available online 27 April 2015

Presented by Michel Talagrand

A B S T R A C T

We obtain explicit formulae, in the form of *regularized multiplicative functionals* related to certain Blaschke products, of the Radon–Nikodym derivatives between reduced Palm measures of all orders for determinantal point processes associated with a large class of weighted Bergman spaces on the disk. Our method also applies to determinantal point processes associated with weighted Fock spaces.

R É S U M É

On obtient des formules explicites, sous forme des fonctionnelles multiplicatives régularisées liées à certains produits de Blaschke, des dérivées de Radon–Nikodym entre toutes les mesures de Palm pour les processus déterminantaux associés aux espaces de Bergman pondérés sur le disque. Notre méthode s'applique également aux processus déterminantaux associés aux espaces de Fock pondérés.

Version française abrégée

Soit $\omega : \mathbb{D} \to \mathbb{R}^+$ un poids sur le disque unité $\mathbb{D}$. Soit $L^2_{\text{hol}}(\mathbb{D}, \omega(z)dz)$ l'espace de Bergman associé et soit K_ω son noyau reproduisant. Notons par $\mathbb{P}_{K_\omega}$ le processus déterminantal induit par K_ω (cf. [6,12]). Étant donné un l-uplet $\mathfrak{p} = (p_1, \cdots, p_l)$ de points distincts dans $\mathbb{D}$, sous certaines conditions sur le poids ω, la limite suivante

E-mail addresses: bufetov@mi.ras.ru (A.I. Bufetov), yqi.qiu@gmail.com (Y. Qiu).

http://dx.doi.org/10.1016/j.crma.2015.03.018
1631-073X/

$$S_{\mathfrak{p}}(\mathcal{Z}) := \lim_{r\to 1^-}\left(\sum_{z\in\mathcal{Z}:|z|\le r}\log\prod_{j=1}^{l}\left|\frac{p_j - z}{1-\bar{p}_j z}\right| - \mathbb{E}_{\mathbb{P}_{K_w}}\sum_{z\in\mathcal{Z}:|z|\le r}\log\prod_{j=1}^{l}\left|\frac{p_j - z}{1-\bar{p}_j z}\right|\right)$$

existe presque sûrement et la fonction $\mathcal{Z}\to e^{2S_{\mathfrak{p}}(\mathcal{Z})}$ est intégrable par rapport à $\mathbb{P}_{K_\omega}$. En revanche, on remarque que le produit $\prod_{z\in\mathcal{Z}}|\frac{p-z}{1-\bar{p}z}|$ diverge presque sûrement pour tout $p\in\mathbb{D}$.

Notre premier résultat (voir Theorem 3.1) est : soit ω un poids sur $\mathbb{D}$ vérifiant certaines conditions (voir inegalité (1)). Pour tout l-uplet $\mathfrak{p}=(p_1,\dots,p_l)\in\mathbb{D}^l$ de points distincts, notons par $\mathbb{P}^{\mathfrak{p}}_{K_\omega}$ la mesure de Palm réduite du processus $\mathbb{P}_{K_\omega}$ correspondant aux points $p_1,\dots,p_l$. Alors, la dérivée de Radon–Nikodym de $\mathbb{P}^{\mathfrak{p}}_{K_\omega}$ par rapport à $\mathbb{P}_{K_\omega}$ est donnée par

$$\frac{d\mathbb{P}^{\mathfrak{p}}_{K_\omega}}{d\mathbb{P}_{K_\omega}}(\mathcal{Z}) = \frac{e^{2S_{\mathfrak{p}}(\mathcal{Z})}}{\mathbb{E}_{\mathbb{P}_{K_\omega}}(e^{2S_{\mathfrak{p}}})}.$$

Il en résulte immédiatement l'équivalence de toutes les mesures de Palm de $\mathbb{P}_{K_\omega}$. Notons que l'équivalence des measures de Palm au cas uniforme $\omega\equiv 1$ est due à Holroyd–Soo [3]. Un corollaire du résultat précédent est la quasi-invariance de la mesure $\mathbb{P}_{K_\omega}$ sous l'action naturelle du groupe $\mathrm{Diff}_c(\mathbb{D})$ de difféomorphismes ayant des supports compacts dans $\mathbb{D}$ sur l'espace de configuration $\mathrm{Conf}(\mathbb{D})$.

On étudie également les espaces de Fock généralisés. Soit $\psi:\mathbb{C}\to\mathbb{R}^+$ et soit G_ψ le noyau reproduisant d'espace de Fock $\mathcal{F}_\psi = L^2_{\mathrm{hol}}(\mathbb{C}, e^{-\psi(z)}\,dz)$. La fonction ψ est supposée de vérifier certaines conditions (voir Theorème 3.2). Alors, pour tout couple $\mathfrak{p}=(p_1,\dots,p_l), \mathfrak{q}=(q_1,\dots,q_l)\in\mathbb{C}^l$ de points distincts, les mesures de Palm réduites $\mathbb{P}^{\mathfrak{p}}_{G_\psi}$ et $\mathbb{P}^{\mathfrak{q}}_{G_\psi}$ sont équivalentes et la dérivée de Radon–Nikodym de $\mathbb{P}^{\mathfrak{p}}_{G_\psi}$ par rapport à $\mathbb{P}^{\mathfrak{q}}_{K_G}$ est calculée explicitement ; notons que le cas gaussien $\psi(z) = |z|^2$ est dû à Osada–Shirai [8].

1. Outline of results

The main purpose of this paper is to announce the equivalence between reduced Palm measures of determinantal point processes associated with a large class of Hilbert spaces of holomorphic functions on a domain or the whole plane $\mathbb{C}$. Two main cases considered in this paper are weighted Bergman spaces on the unit disk $\mathbb{D}$ and weighted Fock spaces on the whole plane $\mathbb{C}$. In the case of Bergman spaces, we obtain explicit formulae of Radon–Nikodym derivatives between reduced Palm measures *of all orders* in the form of certain Blaschke products.

We start with the Bergman case. Let $\omega:\mathbb{D}\to\mathbb{R}^+$ be a weight on $\mathbb{D}$ and let $L^2_{\mathrm{hol}}(\mathbb{D},\omega(z)\,dz)$ be the associated Bergman space. Denote by K_ω the corresponding reproducing kernel. Let $\mathbb{P}_{K_\omega}$ denote the determinantal point process induced by K_ω (cf. [6,12]). Our notation follows that of [1]. Under appropriate conditions on ω, all the reduced Palm measures of $\mathbb{P}_{K_\omega}$ are proved to be equivalent. We obtain explicit formulae for the corresponding Radon–Nikodym derivatives in the form of certain Blaschke products. To have a quick view of the formulae, suppose that $\mathfrak{p}=(p_1,\dots,p_l)\in\mathbb{D}^l$ is any l-tuple of distinct points, then under certain conditions on the weight ω, the limit

$$S_{\mathfrak{p}}(\mathcal{Z}) := \lim_{r\to 1^-}\left(\sum_{z\in\mathcal{Z}:|z|\le r}\log\prod_{j=1}^{l}\left|\frac{p_j - z}{1-\bar{p}_j z}\right| - \mathbb{E}_{\mathbb{P}_{K_w}}\sum_{z\in\mathcal{Z}:|z|\le r}\log\prod_{j=1}^{l}\left|\frac{p_j - z}{1-\bar{p}_j z}\right|\right)$$

exists almost surely and the function $\mathcal{Z}\to e^{2S_{\mathfrak{p}}(\mathcal{Z})}$ is integrable with respect to $\mathbb{P}_{K_\omega}$. Note however that for any $p\in\mathbb{D}$, the product $\prod_{z\in\mathcal{Z}}|\frac{p-z}{1-\bar{p}z}|$ diverges for $\mathbb{P}_{K_w}$-almost every configuration $\mathcal{Z}$.

Our first result says that

$$\frac{d\mathbb{P}^{\mathfrak{p}}_{K_\omega}}{d\mathbb{P}_{K_\omega}}(\mathcal{Z}) = \frac{e^{2S_{\mathfrak{p}}(\mathcal{Z})}}{\mathbb{E}_{\mathbb{P}_{K_\omega}}(e^{2S_{\mathfrak{p}}})}.$$

The Radon–Nikodym derivatives between $\mathbb{P}^{\mathfrak{p}}_{K_\omega}$ and $\mathbb{P}^{\mathfrak{q}}_{K_\omega}$ for tuples $\mathfrak{p}\in\mathbb{D}^l$ and $\mathfrak{q}\in\mathbb{D}^k$ follow immediately. Note that the equivalence of Palm measures in the uniform case $\omega\equiv 1$ is due to Holroyd and Soo [3].

We also consider the generalized Fock spaces. Let $\psi:\mathbb{C}\to\mathbb{R}^+$ and denote $d\nu_\psi(z) = e^{-\psi(z)}dz$. Denote by $\mathcal{F}_\psi = L^2_{\mathrm{hol}}(\mathbb{C}, d\nu_\psi)$ the associated Fock space and let G_ψ be its reproducing kernel. Consider $\mathbb{P}_{G_\psi}$ the determinantal point process induced by G_ψ with respect to measure $d\nu_\psi$. Then under appropriate conditions, we get the equivalence between the reduced Palm measures *of the same order*; the corresponding Radon–Nikodym derivatives are also derived, which in this time have similar formulae as those for the Ginibre point process (corresponding to ψ_2) obtained in [8].

Remark 1. The Radon–Nikodym derivatives between reduced Palm measures of the same orders in the case of Bergman kernel processes are quite different from the formula in Theorem 3.2. Note that the formulae in [7] in the case of Gamma kernel processes, formulae in [1] in the case of determinantal point processes on $\mathbb{R}$ with integrable kernels and formulae in [8] in the case of Ginibre point process are all similar to the formula in Theorem 3.2.