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An optimization approach for the satisfiability problem

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Abstract We describe a new approach for solving the satisfiability problem by geometric programming. We focus on the theoretical background and give details of the algorithmic procedure. The algorithm is provably efficient as geometric programming is in essence a polynomial problem. The correctness of the algorithm is discussed. The version of the satisfiability problem we study is exact satisfiability with only positive variables, which is known to be NP-complete.

1. Introduction

The satisfiability problem is still the most important open problem in computer science. Many scientific disciplines highly depend on efficient solutions of this core problem. These span computer science itself, mathematical logic, pure and applied mathematics, physics, chemistry, economics, and engineering, just to mention some. Approximations and heuristics are more than welcome in applications, since

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they oftentimes provide efficient solutions for special cases. Theoretically, however, the computer science community is still lacking a good understanding of the computational difficulties of this problem though many facts about it are already known. Complexity theory tells us that the actual problem is related to the $P = NP$? question. Indeed, this question is now becoming a *complex* offered to us by complexity theory. Much effort to solve it has been in vain so far. Regrettably, no real progress in the discipline of computer science can be made until the problem is solved. Most (senior) computer scientists today rather believe that P is not equal to NP. That is, no matter how much we try, the satisfiability problem, as a prominent NP-complete problem, will persist as a challenging problem with exponential known deterministic worst-case complexity.

It is also disappointing that the NP-complete problems are too many and essential in real applications. It is rather simple to find a new NP-complete problem: just try to solve one of them and describe the main difficulty you encounter as a new problem if possible. So, adding a new problem to the list is relatively easy, but removing one from the list seems to be very hard. The reason is that the list of NP-complete problems degenerates to the empty list as soon as one of the problems is discovered to be outside the list, that is, as soon as one of the problems is proved to be P-complete. This very fact is the main achievement of the theory of NP-completeness.

Prevalent approaches for the satisfiability problem have concentrated so far on: logical manipulation of formulas, graph-theoretic algorithms, probabilistic algorithms, and mathematical optimization. Two methods based on logical manipulations are worth mentioning: the Davis Putnam (DPLL) procedure and the method of resolution and reduction. DPLL (Bacchus, 2002) proved extremely efficient for most SAT instances arising in practice. Propositional resolution (Robinson, 1965) is a well-understood powerful method. Propositional reduction (Noureddine, submitted for publication) is also powerful and works well in conjunction with resolution. Graph-theoretic algorithms (Aspvall et al., 1979) are extremely helpful for analyzing special algorithms (e.g. algorithms for 2SAT). Probabilistic algorithms (Schoening, 1999) are also of extreme importance in practice and might deliver the best results for complex instances. Finally, the method of mathematical optimization (Fletcher, 1987) treats the satisfiability problem as an optimization problem and not as a decision problem. The method is very promising in a real setting though implementing reliable optimization code is a very challenging task.

We favor in this paper the method of mathematical optimization. In another paper, we focused on formulating the satisfiability problem as non-convex, exact, exterior, penalty-based problem with a coercive objective function. The method focused on exact satisfiability (XSAT), which is NP-complete. The method falls into the category of approximation schemes for solving the satisfiability problem and is sub-optimal and partially heuristic in nature. In this paper, we treat the problem by way of geometric programming. We still focus on XSAT or more precisely on 3XSAT, which has the following properties: