COMPUTER METHODS AND PROGRAMS IN BIOMEDICINE 94 (2009) 232–238

journal homepage: www.intl.elsevierhealth.com/journals/cmpb

A mathematical function to evaluate surgical complexity of cleft lip and palate

M.R. Ortiz-Posadas[a,*], L. Vega-Alvarado[b], B. Toni[c]

[a] Departamento de Ingeniería Eléctrica, Universidad Autónoma Metropolitana-Iztapalapa, Av. San Rafael Atlixco #186, Col. Vicentina, C.P. 09340 Iztapalapa, México, D.F., Mexico

[b] Centro de Ciencias Aplicadas y Desarrollo Tecnológico, Universidad Nacional Autónoma de México, Mexico

[c] Department of Mathematics and Computer Science, Virginia State University, Petersburg, VA, USA

ARTICLE INFO

Article history:
Received 6 May 2008
Received in revised form
1 December 2008
Accepted 29 December 2008

Keywords:
Cleft palate
Surgical complexity
Similarity function
Mathematical modeling

ABSTRACT

The objective of this work is to show the modeling of a similarity function adapted to the medical environment using the logical–combinatorial approach of pattern recognition theory, and its application comparing the condition of patients with congenital malformations in the lip and/or palate, which are called *cleft-primary palate* and/or *cleft-secondary palate*, respectively. The similarity function is defined by the comparison criteria determined for each variable, taking into account their type (qualitative or quantitative), their domain and their initial space representation. In all, we defined 18 variables, with their domains and six different comparison criteria (fuzzy and absolute difference type). The model includes, further, the importance of every variable as well as a weight which reflects the surgical complexity of the cleft. Likewise, the usefulness of this function is shown by calculating the similarity among three patients. This work was developed jointly with the Cleft Palate Team at the Reconstructive Surgery Service of the Pediatric Hospital of Tacubaya, which belongs to the Health Institute of the Federal District in Mexico City.

1. Introduction

The concept of an analogy is a fundamental methodological tool in soft sciences, such as Medicine. For many workers attempting mathematical modeling, the likelihood between two objects may be represented using a function of distance (a norm) since closeness and likelihood have generally been treated as synonyms. That is, two objects are more alike the closer they are found from each other and, given this, it is possible to agree if some details are specified, such as the space of representation of the objects, the kind of variables (qualitative or quantitative) which describe them, their domains, the comparison criteria for their values, etc. Likewise, it is important to consider the way in which full object descriptions are attempted. In this sense, it is important to distinguish between likelihood and closeness in those cases where these terms are not synonyms. Therefore, we have to exercise care in the use of a metric when attempting to model the likelihood between objects.

The objective of this work is to show the modeling of a similarity function found with the logical–combinatorial approach of pattern recognition theory [1] in medicine environment. The clinical problem consists of congenital malformations in the lip and/or palate, which are called *cleft-primary palate* and/or *cleft-secondary palate*, respectively [2]. We develop the model of a similarity function for comparing the condition of patients with these kinds of malformations. Such function is defined by the comparison criteria determined for each variable taking

* *Corresponding author*. Tel.: +52 55 5804 4904; fax: +52 55 5804 4628.
E-mail address: posa@xanum.uam.mx (M.R. Ortiz-Posadas).

0169-2607/$ – see front matter
doi:10.1016/j.cmpb.2008.12.008

into account their type (qualitative or quantitative), their domain and their initial space representation. Likewise, the usefulness of this function is shown by calculating the similarity among four patients. This work was developed jointly with the Cleft Palate Team at the Reconstructive Surgery Service of the Pediatric Hospital of Tacubaya, which belongs to the Health Institute of the Federal District in Mexico City.

2. Mathematical model

2.1. *Definitions and notations*

Let $\mathbf{O} = \{O_1, \ldots, O_m\}$ be a finite set of m objects, each object is described in terms of the finite set of variables $\mathbf{X} = \{x_1, \ldots, x_n\}$, where each variable x_i, $i = 1, \ldots, n$ is defined on its domain

$M_i = \{m_{i1}, m_{i2}, \ldots\} U \{*\}$ where $*$ denotes absence of information.

Definition 1. Let the *initial space representation* (ISR) be the object space representation defined by the Cartesian product of M_i sets:

$$I(O) = (x_1(O), \ldots, x_n(O)) \in (M_1 \times \ldots \times M_n)$$

where $I(O)$ is the object description of O in terms of the variables x_i, $i = 1, \ldots, n$.
$x_i(O)$ is the value taken by the variable x_i in the object O.

Remark. We do not assume any algebraic or topologic structure over ISR. That is, there is no defined *a priori* norm or logic or algebraic operation over M_i. But this does not mean that they cannot be present in ISR. Some times one can consider a function which does not satisfy the norm properties over M_i (over ISR). Asymmetric functions [3], non-redundant asymmetric functions [4] or symmetric functions which do not satisfy the triangle inequality [5] may be considered as well.

Definition 2. Let $\omega \subseteq X$ be a support set, where $\omega \neq \{\emptyset\}$. A system of support sets is defined as $\Omega = \{\omega_1, \ldots, \omega_s\}$. By ωO we denote the ω-part of O formed by the variables $x_j \in \omega_m$, $m = 1, \ldots, s$.

Definition 3. Let $C = \{C_1, \ldots, C_n\}$ be a set of functions called *comparison criteria* for each variable $x_i \in \mathbf{X}$ such as: C_i: $M_i \times M_i \to \Delta_i$; $i = 1, \ldots, n$ where Δ_i can be of any nature; it is an ordered set and can be finite or infinite [6].

Remark *([7])*. Comparison criteria can denote similarity or difference. For the sake of illustration, we present the following examples of an ordered set Δ_t.

(a) If $\Delta_i = [0,1]$, the one value represents maximum similarity/difference and the zero value the minimum. Any intermediate value represents a grade of similarity/difference between compared values.
(b) If the comparison criterion is finite valued, that is: $\Delta_i = \{0, 1, 2, \ldots, m\}$, it is not difficult to transform Δ_i into: $\Delta_i = \left\{0, 1/m, 2/m, \ldots, (m-1)/m, 1\right\}$. Obviously this is a subset of the interval [0,1]. Then, in this case, the intermediate values Δ_i also represent a grade of similarity/difference between compared values.

The characteristics of each comparison criterion (C_i) depend on the problem that has been modeled. However, it is important to remark that every C_i is designed individually to reflect the nature and interpretation of each feature x_i. In this sense, the set C permits differentiation and non-uniform treatment of the features that describe the objects. Furthermore, it gives the possibility of "absent information" in some feature values in the objects descriptions. It is important to mention that all comparison criteria must be defined jointly with the *expert* in order to incorporate his/her expertise in the problem modeling. This expert can be formally assumed to play the role of an *Oracle* in Turing Machines. Intuitively, an Oracle TM is a model of computation where a Turing Machine is "endowed" with the ability to decide membership in some language A [8]. In the context of our clinical problem, this Oracle will be the reconstructive and plastic face surgeon with her/his knowledge and expertise, with the ability to provide the entire criterion about cleft lip and palate for the problem modeling.

The analogy between two objects is formalized by means of the concept of *similarity function*. This function is based on the *comparison criterion* C_i generated for each variable x_i. It is important to mention that the similarity function can evaluate the similarity or difference between two objects, i.e., between their descriptions.

Definition 4. Let β: $(M_i \times M_i)^2 \to \Delta$ be the *similarity function*, where Δ (as in the comparison criterion function) can be of any nature; it is an ordered set and can be finite or infinite. For $I(O_i)$ and $I(O_j)$ being two object descriptions in the domain $(M_1 \times \ldots \times M_n)$, β is defined by:

$$\beta(I(O_i), I(O_j)) := \begin{cases} \beta((C_1(x_1(O_i), x_1(O_j)), \ldots, C_n(x_n(O_i), x_n(O_j)))), & \text{if } C_i \text{ denotes similarity} \\ 1 - \beta((C_1(x_1(O_i), x_1(O_j)), \ldots, C_n(x_n(O_i), x_n(O_j)))), & \text{if } C_i \text{ denotes difference} \end{cases}$$

Remark. Both, the comparison criterion and the similarity function, are not necessarily symmetric [3] and, in general, these functions are not defined as positive, they do not have to satisfy the triangle inequality, and there is no *a priori* metric considered.

Definition 6. Let β_ω be a *partial* similarity function defined by:

$$\beta_\omega(I(O_i), I(O_j)) = 1 - \sum_{x_t \in \omega} \rho_t C_t(x_t(O_i), x_t(O_j))$$

where ω represents a support set. ρ_t is the relevance parameter associated to each variable x_t defined by the expert.

The analogy concept is a fundamental methodological tool in soft sciences, as medicine. In general, physicians reach their conclusion on the basis of analogies found by accumulated knowledge through their experiences and observations. The analogy concept is presented in almost all the reasoning and conclusions of the medicine specialists. In this sense, to evaluate the similarity between patients, considering their