ELSEVIER

journal homepage: www.intl.elsevierhealth.com/journals/cmpb

Boundary element method for prediction of hardness of dentin from punch–nano-indentation test

H. Gun *

Usak University, Usak, Turkey

ARTICLE INFO

Article history:
Received 12 January 2007
Received in revised form
7 March 2007
Accepted 7 March 2007

Keywords:
Boundary element
Plasticity
Nano-indentation
Hardness

ABSTRACT

In this paper, a quadratic boundary element formulation for prediction of hardness of dentin from punch–nano-indentation test is presented. BE contact formulation is given. The initial strain formulation and von Mises yield criteria are employed to cover plastic deformation. The dentin is assumed to be isotropic, homogenous and elastic-perfectly plastic material. The load versus displacement is obtained during loading–unloading sequence for different yield strength and elastic modulus. Hardness and equivalent residual stresses are plotted for different elastic modulus depending on yield strength. BE method is shown to be an alternative accurate computational tool for simulated nano-indentation tests.

1. Introduction

The boundary element (BE) method is well established as an accurate numerical tool, particularly well suited for linear elastic problems. Due to its high resolution of stresses on the surface, the BE approach has been shown to be accurate in problems involving stress concentration, fracture mechanics and contact analysis. However, its extension to non-linear problems including material and geometric non-linearity is not widespread and is under-developed when compared to the finite element (FE).

Nano-indentation is a powerful process for determining the mechanical properties of materials of small dimensions. However, experimental methods are not convenient for determining yield strength and yield curves. Therefore, it requires computational approaches. FE analysis of sphere-nano-indentation is studied [1] with FE mesh design of 4950 4-noded axisymmetric elements. In this work, there is a large deviation between loading and unloading curves. Moreover, unloading curves are almost a straight line. In their conclusion, this may be due to different yield strength or constitutive model, like perfect elastic–plastic one employed in their FE analysis. However, it may not be one of these. It might be the way FE formulation, employed in their analysis, handles contact problems with plasticity.

In this paper, a BE formulation based on an initial strain approach is presented for such applications. Isoparametric quadratic elements are employed for the line elements and surface cells. The objective of this paper is to present the BE method as an alternative accurate approach for simulated nano-indentation tests.

2. BE analytical formulations

By considering the plastic strain increments as initial strain increments then modifying Betti's reciprocal theorem to include plasticity, the pseudo-boundary integral equation for

* Tel.: +90 276 2634195; fax: +90 276 2634196.
E-mail address: hgun@aku.edu.tr.
0169-2607/$ – see front matter © 2007 Elsevier Ireland Ltd. All rights reserved.
doi:10.1016/j.cmpb.2007.03.003

initial strain approach can be written (by neglecting body forces) as follows:

$$C_{ij}(P)\dot{u}_i(P) = -\int_S T_{ij}(P,Q)\dot{u}_i(Q)\,dS_Q + \int_S U_{ij}(P,Q)\dot{t}_i(Q)\,dS_Q + \int_A (W_{kij}(P,q) + \bar{W}_{kij}(P,q))\dot{\varepsilon}^p_{ij}(q)\,dA(q) \quad (1)$$

In this expression U_{ij}, T_{ij} are the second-order displacement and traction tensors in the i direction at the field point Q or q due to an orthogonal unit load at the variable point P or p in the j direction. u_i, t_i and ε^p_i are displacement, traction and plastic strain respectively. The dots represent incremental variables for time-independent plasticity. Capital letters are used to indicate that the point concerned lies on the boundary S. Capital letter A represents the solution domain. C_{ij} is the free-term tensor, whose components depend on the geometry, and W_{kij} are the corresponding stress components. These tensors can be derived from the fundamental solution to Kelvin's problem in two dimensions and the auxiliary tensor, $\bar{W}_{kij}$, is given as follows:

$$\bar{W}_{kij}(p,q) = \frac{\nu}{2\pi(1-\nu)}\delta_{ij}\frac{1}{r}\frac{\partial r}{\partial x_k} \quad \text{(plane strain)},$$
$$\bar{W}_{kij}(p,q) = 0 \quad \text{(plane stress)} \quad (2)$$

The correct expressions of the plastic deformation for interior nodes, convected differentiation of the related domain integrals are employed and at internal points the total plastic strains can then be written as follows:

$$\dot{\varepsilon}_{ij}(p) = -\int_S S^{\varepsilon}_{kij}(p,Q)\dot{u}_k(Q)\,dS(Q) + \int_S D^{\varepsilon}_{kij}(p,Q)\dot{t}_k\,dS(Q) + \int_A W^{\varepsilon}_{ijkh}(p,q)\dot{\varepsilon}^P_{kh}\,dA(q) + \int_A \bar{W}^{\varepsilon}_{ijkh}(p,q)\,dA(q) + F^{\varepsilon}_{ij}(\dot{\varepsilon}^P_{kh}(p)) \quad (3)$$

in which D^{ε}_{kij}, S^{ε}_{kij} and W^{ε}_{ijkh}, are the derivatives of the aforementioned fundamental solutions [2]. The auxiliary tensor (from plastic strain terms in the out-of-plane direction), W^{ε}_{ijkh} is given as:

$$\bar{W}^{\varepsilon}_{ijkh}(p,q) = -\frac{\nu}{4\pi(1-\nu)}\left(\frac{1}{r^2}\right)\left(\delta_{ij}\delta_{kh} - 2\delta_{ij}\frac{\partial r}{\partial x_k}\frac{\partial r}{\partial x_h}\right)$$
$$\text{(plane strain)}, \quad \bar{W}^{\varepsilon}_{ijkh}(p,q) = 0 \quad \text{(plane stress)} \quad (4)$$

The integral free-term, F^{ε}_{ij} depends on the plastic deformation at the load point and it is given by:

$$F^{\varepsilon}_{ij}(\dot{\varepsilon}^P_{kh}(p)) = \frac{3-4\nu}{4(1-\nu)}\dot{\varepsilon}^P_{kh}(p) - \frac{1}{8(1-\nu)}\delta_{kh}\dot{\varepsilon}^P_{mm}(p) \quad \text{(plane stress)},$$
$$F^{\varepsilon}_{ij}(\dot{\varepsilon}^P_{kh}(p)) = \frac{3-4\nu}{4(1-\nu)}\dot{\varepsilon}^P_{kh}(p) - \frac{1-4\nu}{8(1-\nu)}\delta_{kh}\dot{\varepsilon}^P_{mm}(p) \quad \text{(plane strain)} \quad (5)$$

For a material obeying the von Mises yield criterion and linear isotropic hardening, the plastic stain increments are given by the following incremental elastoplastic flow rule:

$$\dot{\varepsilon}^p_{ij} = \frac{3}{2}\left(\frac{\dot{S}_{kl}\dot{\varepsilon}_{kl}}{1+H'/3\bar{\mu}}\right)\frac{\dot{S}_{ij}}{(\dot{\sigma}_{eq})^2} \quad (6)$$

in which $\dot{S}_{ij}$ and $\dot{\sigma}_{eq}$ denote the current deviatoric stress tensor and the equivalent stress respectively. $\dot{\varepsilon}_{ij}$ is the total strain increments, $\bar{\mu}$ the shear modulus and H' represents the plastic modules.

3. Numerical implementation

The detailed formulation of the BE method for the plastic analysis using isoparametric quadratic elements is well covered in the literature and will be summarized here. Since interior modelling is required, both boundary elements and domain cells are necessary to perform the integrals. For the boundary element, the geometry can be described in terms of quadratic shape functions in a local co-ordinate axes system as follows:

$$x_i(\xi) = \sum_{c=1}^{3} N_c(\xi)(x_i)_c \quad (7)$$

where N_c is the quadratic shape function and ξ is the local co-ordinate. Similarly, the displacement and traction vectors can be expressed in terms of quadratic shape functions. For the interior cells, the geometry can be defined in terms of quadratic shape functions which are described in local co-ordinates ξ_1 and ξ_2 as follows:

$$x_i(\xi_1,\xi_2) = \sum_{c=1}^{8} N_c(\xi_1,\xi_2)(x_i)_c \quad (8)$$

Similarly, the displacement increments can be expressed in the domain cells. Therefore, in a discretised form, for each body in contact, the elastoplastic boundary integral equation in the initial strain approach (by neglecting body forces) can be written as follows:

$$C_{ij}\dot{u}_j(P) + \sum_{m=1}^{M}\sum_{c=1}^{3}\dot{u}_j(Q)\int_{-1}^{+1} T_{ij}(P,Q)N_c(\xi)J(\xi)\,d\xi$$
$$= \sum_{m=1}^{M}\sum_{c=1}^{3}\dot{t}_j(Q)\int_{-1}^{+1} U_{ij}(P,Q)N_c(\xi)J(\xi)\,d\xi$$
$$+ \sum_{m=1}^{D}\sum_{c=1}^{8}\dot{\varepsilon}^p_{ij}(q)\int_{-1}^{+1}\int_{-1}^{+1} W_{ijk}(p,q)N_c(\xi_1,\xi_2)J(\xi_1,\xi_2)\,d\xi_1\,d\xi_2$$
$$+ \sum_{m=1}^{D}\sum_{c=1}^{8}\dot{\varepsilon}^p_{ij}(q)\int_{-1}^{+1}\int_{-1}^{+1} \bar{W}_{ijk}(p,q)N_c(\xi_1,\xi_2)J(\xi_1,\xi_2)\,d\xi_1\,d\xi_2 \quad (9)$$

where P denotes the node where the integration is performed, Q indicates the cth node of the mth boundary element and q indicates the cth node of the mth domain cell.

It should be noted that the integration process is performed separately for each domain in contact. A set of linear algebraic