Available online at www.sciencedirect.com

ScienceDirect

Indagationes Mathematicae 27 (2016) 835–849

indagationes mathematicae

www.elsevier.com/locate/indag

Hybrid functions of Bernstein polynomials and block-pulse functions for solving optimal control of the nonlinear Volterra integral equations

Farshid Mirzaee*, Seyede Fatemeh Hoseini

Faculty of Mathematical Sciences and Statistics, Malayer University, P. O. Box 65719-95863, Malayer, Iran

Received 5 June 2015; received in revised form 30 September 2015; accepted 4 March 2016

Communicated by H. Woerdeman

Abstract

The main purpose of this paper is to approximate the solution of the optimal control problem for systems governed by a class of nonlinear Volterra integral equations. In order to do this, we use combination of Bernstein polynomials (BPs) and block-pulse functions (BPFs) on the interval $[0, 1)$ for converting this problem to an optimization problem that can be solved easily by mathematical programming techniques. Also, the convergence of the proposed method is discussed. Furthermore, in order to show the accuracy and reliability of the proposed method, the new approach is applied to some practical problems.

Keywords: Optimal control problem; Volterra integral equation; Block-pulse functions; Bernstein polynomials; Collocation method

1. Introduction

Consider a class of Volterra integral equations (VIEs)

$$x(t) = y(t) + \lambda \int_0^t k(s, t, u(s))x(s)ds, \tag{1}$$

* Corresponding author. Tel.: +98 81 32355466; Fax: +98 81 32355466.
E-mail addresses: f.mirzaee@malayeru.ac.ir, fa_mirzaee@yahoo.com (F. Mirzaee), fatemeh.hoseini@stu.malayeru.ac.ir (S.F. Hoseini).

http://dx.doi.org/10.1016/j.indag.2016.03.002

where the continuous real-valued functions $x(s)$ and $u(s)$ are the state of the controlled system (trajectory function) and the control function, respectively, $k \in C([0,\ T] \times [0,\ T] \times \Omega)$, $y \in C[0,\ T]$ and $\lambda \in R$, in which $\Omega \subseteq R^n$ is a compact set. It is known that for a given continuous control $u(t)$, the VIE has a unique and continuous solution. The problem of finding numerical solution for such integral equations is one of the oldest problems in applied mathematics and many computational methods are proposed in this area [3,4,13,25]. But not much work has been reported on the optimal control problems (OCPs) of nonlinear systems. Optimal control problem is determining the optimal control $u(t)$ and the corresponding optimal state $x(t)$ satisfying (1) while minimizing the cost functional

$$J(x,u) = \int_0^T \Psi(t, x(t), u(t))dt, \tag{2}$$

where $\Psi \in C([0,\ T] \times R \times \Omega)$. In the rest of the paper, it is supposed $T = 1$ and $\Psi(t, x(t), u(t)) = u^2(t) + x^2(t) + f(t)x(t) + g(t)u(t)$ where $f(t)$ and $g(t)$ are real functions in $L^2[0, 1]$. Also, it is assumed that the optimal control of this problem is unique.

The classical theory of optimal control was originally developed to deal with systems of controlled ordinary differential equations [20]. Due to literature [16,22], a wide class of controlled systems in economics, biology, epidemiology, and memory effects can be described by Volterra integral equations instead of ordinary differential equations. Because of the complexity of most applications, optimal control problems are most often solved numerically. Numerical methods for solving optimal control problems date back to the 1950s with the works of Bellman [5,8,6, 10,7,9].

There are different techniques for solving the optimal control problem governed by Volterra integral equations. Recently, homotopy perturbation method (HPM) [12], Chebyshev polynomials [38], Legendre polynomials [36] and a hybrid method based on steepest descent and Newton methods [30] have been used for solving OCPs governed by VIEs. In [34] the author developed a general framework for nonlinear optimal control problem by employing BPFs. Using quasilinearization and Chebyshev polynomials, a numerical method to solve nonlinear optimal control problem was presented in [21]. A class of computational methods based on pseudo spectral approximations was presented in [33] for solving a wide variety of optimal control problems. BPFs were used in [14] to solve linear and nonlinear optimal control problems with constraints. Also, the necessary and sufficient conditions on the existence solution optimal control of Volterra integral equations have been considered in [37]. Existence and uniqueness of the solution of the optimal control of systems governed by VIEs can be found in [2].

In this paper, we propose to study optimal control of systems governed by some classes of nonlinear Volterra integral equations which can be described by (2) subject to state dynamics (1). For this purpose, we use hybrid functions of Bernstein polynomials and block-pulse functions companied with the sequential quadratic programming (SQP) algorithm. Bernstein polynomials are quite easy to write down: the coefficients can be obtained from Pascal's triangle. It can easily be shown that each of the BPs is positive and also the sum of all the BPs is unity. Bernstein polynomials form a complete basis over the interval [0, 1]. It is easy to show that any given polynomial of degree M can be expressed in terms of linear combination of the basis functions and only a small number of BPs are needed to obtain a satisfactory result. Block-pulse functions are studied by many authors and applied for solving different problems [14,34]. The most important properties of BPFs are disjointness, orthogonality, and completeness. BPFs are very common in use and they improve the rate of convergence in some other methods such as hybrid methods.