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On a characterization of the structured Wolf, Schechter and Browder essential pseudospectra

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Abstract

In this paper, we concern ourselves with essential spectra of an operator A which is subjected to structured perturbation of the form $A \longrightarrow A + CDB$ where B, C are given bounded operators and D is unknown disturbance operator that satisfies $\|D\| < \varepsilon$ for a given $\varepsilon > 0$. Thereby, we investigate the structured essential pseudospectra of the sum of two operators acting on a Banach space. Furthermore, we establish characterizations of the structured Schechter essential pseudospectrum of a closed, densely defined linear operator.

Keywords: Fredholm perturbation; Wolf, Schechter and Browder essential spectra; Structured pseudospectra

1. Introduction

Let X be a Banach space. By $\mathcal{C}(X)$ we denote the set of all closed, densely defined linear operators on X, by $\mathcal{L}(X, Y)$ the set of all bounded linear operators from X into Y. The subset of all compact operators of $\mathcal{L}(X)$ is designated by $\mathcal{K}(X)$. For $A \in \mathcal{C}(X)$, we let $\sigma(A)$, $\rho(A)$, $R(A)$ and $N(A)$ denote the spectrum, resolvent set, the range and the null space of A, respectively. The

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nullity, $\alpha(A)$, of A is defined as the dimension of $N(A)$ and the deficiency, $\beta(A)$, of A is defined as the codimension of $R(A)$ in X. The set of Fredholm operators is defined by

$$\Phi(X) = \Big\{A \in \mathcal{C}(X);\ \alpha(A) < \infty,\ \beta(A) < \infty,\ R(A) \text{ is closed in } X\Big\}.$$

If $A \in \Phi(X)$, the number $i(A) = \alpha(A) - \beta(A)$ is called the index of A.

In this paper we are concerned with the following essential spectra of a closed, densely defined linear operator A:

$$\sigma_{e_4}(A) := \mathbb{C} \setminus \Big\{\lambda \in \mathbb{C} \text{ such that } \lambda - A \in \Phi(X)\Big\} = \mathbb{C} \setminus \Phi_A.$$

$$\sigma_{e_5}(A) := \mathbb{C} \setminus \Big\{\lambda \in \Phi_A \text{ such that } i(\lambda - A) = 0\Big\} = \mathbb{C} \setminus \rho_5(A).$$

$$\sigma_{e_6}(A) := \mathbb{C} \setminus \Big\{\lambda \in \rho_5(A) \text{ such that all scalars near } \lambda \text{ are in } \rho(A)\Big\} = \mathbb{C} \setminus \rho_6(A).$$

The subset $\sigma_{e_4}(.)$ is the Wolf essential spectrum [23], $\sigma_{e_5}(.)$ is the Schechter essential spectrum [8–11] and $\sigma_{e_6}(.)$ denotes the Browder essential spectrum [8,12].

It is well known that if A is a self-adjoint operator on a Hilbert space, then the essential spectrum except isolated eigenvalues of finite algebraic multiplicity (see, for instance, [17]).

Structured pseudospectra are very useful tools in control theory and other fields [7]. In [7], D. Hinrichsen and B. Kelb presented a graphical method to determine and visualize the spectral value sets of a matrix A.

We refer to E. B. Davies [4] which defined the structured pseudospectra, or spectral value sets of a closed densely defined linear operator A on X by

$$\sigma(A, B, C, \varepsilon) = \bigcup_{\|D\| < \varepsilon} \sigma(A + CDB),$$

where $B \in \mathcal{L}(X, Y)$ and $C \in \mathcal{L}(Z, X)$.

In [6], A. Elleuch and A. Jeribi introduced and characterized the structured S-pseudospectra of a closed densely defined linear operator. Furthermore, we defined for the first time the notion of the structured S-essential pseudospectra and we proved their stability by some class of perturbation.

In this paper, we present the set of all complex numbers to which at least one essential spectrum of an operator A can be shifted by perturbation of the form $A \rightsquigarrow A + CDB$, where C and B are fixed bounded operators and the unknown disturbance operator D satisfies $\|D\| < \varepsilon$ for a given $\varepsilon > 0$ (see Definition 3.1). When dealing with structured essential pseudospectra of linear operators on a Banach space, one of the main problems consists in studying the structured essential pseudospectra of the sum of two operators.

In [21], the authors showed that if A_1 and A_2 are respectively closed and bounded operators which commute modulo the compact operators, then they obtained under certain condition a relationship between the Wolf essential spectrum of their sum and the sum of their Wolf essential spectra.

Recently, in [8] this analysis was extended to various essential spectra. More precisely, F. Abdmouleh, S. Charfi and A. Jeribi proved in [1] that if A_1 and A_2 commute modulo the Fredholm perturbations, then they have under some conditions the following result:

$$\sigma_{e_i}(A_1 + A_2) \subset \sigma_{e_i}(A_1) + \sigma_{e_i}(A_2); \quad i = 4, 5, 6.$$