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New description of the structured essential pseudospectra

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Abstract

In this paper, we outline a new approach to the study of structured Schechter and structured Browder essential pseudospectra of closed densely defined linear operators on infinite dimensional Banach spaces via the concept of the measure of noncompactness and the measure of non strict-singularity. Furthermore, we investigate the structured Wolf, structured Schechter and structured Browder essential pseudospectra of the sum of two bounded operators.

Keywords: structured Wolf; Schechter and Browder essential pseudospectra; Measure of noncompactness; Measure of non strict-singularity

1. Introduction

Let X be an infinite-dimensional Banach space. By an operator A from X into a Banach space Y, we mean a linear operator with domain $\mathcal{D}(A) \subset X$ and range $R(A) \subset Y$. By $\mathcal{L}(X, Y)$ we denote the set of all bounded linear operators from X into Y and by $\mathcal{C}(X)$ the set of all closed, densely defined linear operators from X into itself. The set of all compact operators of $\mathcal{L}(X)$ is denoted by $\mathcal{K}(X)$. For $A \in \mathcal{C}(X)$, we let $\sigma(A)$, $\rho(A)$, and $N(A)$ denote the spectrum, resolvent

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set, and the null space of A, respectively. The nullity, $\alpha(A)$, of A is defined as the dimension of $N(A)$ and the deficiency, $\beta(A)$, of A is defined as the codimension of $R(A)$ in X. The set of upper semi-Fredholm operators is defined by

$$\Phi_+(X) := \Big\{A \in \mathcal{C}(X) \text{ such that } \alpha(A) < \infty,\ R(A) \text{ is closed in } X\Big\},$$

and the set of lower semi-Fredholm operators is defined by

$$\Phi_-(X) := \Big\{A \in \mathcal{C}(X) \text{ such that } \beta(A) < \infty,\ R(A) \text{ is closed in } X\Big\}.$$

Operators in $\Phi_\pm(X) := \Phi_+(X) \cup \Phi_-(X)$ are called semi-Fredholm operators on X while $\Phi(X) := \Phi_+(X) \cap \Phi_-(X)$ denotes the set of Fredholm operators on X. The index of an operator $A \in \Phi_\pm(X)$ is $i(A) := \alpha(A) - \beta(A)$.

The set of Fredholm perturbations was introduced and investigated in [11] and is denoted by

$$\mathcal{F}(X) := \Big\{F \in \mathcal{L}(X) \text{ such that } A + F \in \Phi(X) \text{ whenever } A \in \Phi(X)\Big\}.$$

Likewise, it is proved in [19] that

$$\mathcal{K}(X) \subset \mathcal{F}(X) \subset \mathcal{R}(X),$$

where

$$\mathcal{R}(X) := \Big\{A \in \mathcal{L}(X) \text{ such that } \lambda - A \in \Phi(X) \text{ for each } \lambda \neq 0\Big\}$$

which is the class of all Riesz operators. For more information on the family of Riesz operators we refer to [17].

For $A \in \mathcal{C}(X)$, we are concerned with the following essential spectra:

$$\sigma_{e_4}(A) := \mathbb{C} \setminus \Phi_A \quad \text{and} \quad \sigma_{e_5}(A) := \mathbb{C} \setminus \rho_5(A),$$

where

$$\Phi_A := \Big\{\lambda \in \mathbb{C} \text{ such that } \lambda - A \in \Phi(X)\Big\} \quad \text{and}$$

$$\rho_5(A) := \Big\{\lambda \in \Phi_A \text{ such that } i(\lambda - A) = 0\Big\}.$$

The subset $\sigma_{e_4}(.)$ is the Wolf essential spectrum [15,22] and $\sigma_{e_5}(.)$ is the Schechter essential spectrum [15,13,14,12].

To discuss ascent and descent one must consider the case in which $\mathcal{D}(A)$ and $R(A)$ are in the same linear space X. So, we can define the iterates A^2, A^3,... of A. If $n > 1$, $\mathcal{D}(A^n)$ is the set $\Big\{x : x, Ax, \ldots, A^{n-1}x \in \mathcal{D}(A)\Big\}$, and $A^n x = A(A^{n-1}x)$. Obviously, $N(A^n) \subset N(A^{n+1})$ and $R(A^{n+1}) \subset R(A^n)$ for all $n \geq 0$ with the convention $A^0 = I$ (the identity operator on X). Thus, if $N(A^k) = N(A^{k+1})$ (resp. $R(A^k) = R(A^{k+1})$), then $N(A^n) = N(A^k)$ (resp. $R(A^n) = R(A^k)$) for all $n \geq k$. Then, the smallest nonnegative integer n such that $N(A^n) = N(A^{n+1})$ (resp. $R(A^n) = R(A^{n+1})$) is called the ascent (resp. the descent) of A, and denoted by $a(A)$ (resp. $d(A)$). An operator $A \in \mathcal{C}(X)$ is called Browder if $A \in \Phi(X)$, $i(A) = 0$, $a(A) < \infty$ and $d(A) < \infty$. In case where n does not exist, we define $a(A) = \infty$ (resp. $d(A) = \infty$).