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Dynamics of composition operators on weighted Bergman spaces

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Abstract

We discuss the frequent hypercyclicity and hypercyclicity of multiples of the linear fractional composition operators on A^p_α and give equivalent conditions of the frequent hypercyclicity and hypercyclicity. We also characterize disjoint hypercyclicity and disjoint mixing behavior of finitely many linear fractional composition operators on the weighted Bergman spaces A^p_α.

Keywords: Frequently hypercyclic; Disjoint hypercyclic; Composition operators; Weighted Bergman spaces

1. Introduction

Let $\mathbb{D}$ be the open unit disk of the complex plane $\mathbb{C}$, and $d\nu$ denote the Lebesgue measure on $\mathbb{D}$ normalized so that $\nu(\mathbb{D}) = 1$. Denote by $H(\mathbb{D})$ the space of all holomorphic functions on $\mathbb{D}$ and $S(\mathbb{D})$ the collection of all the holomorphic self-maps of $\mathbb{D}$.

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For $\alpha > -1$ and $1 < p < \infty$, the weighted Bergman space A^p_α consists of analytic functions f such that

$$\|f\|^p = \int_D |f(z)|^p \, d\nu_\alpha(z) < \infty,$$

where $d\nu_\alpha$ on $\mathbb{D}$ is defined by

$$d\nu_\alpha = (\alpha+1)\left(1-|z|^2\right)^\alpha d\nu(z),$$

and $\nu_\alpha(\mathbb{D}) = 1$. Under the norm $\|.\|$, A^p_α is a separable infinite dimensional Banach space, since the set of polynomials is dense in A^p_α. By Theorem 2.12 in [16], we know that the dual space of A^p_α is A^q_α under the integral pairing

$$\langle f, g\rangle = \int_{\mathbb{D}} f(z)\overline{g(z)} d\nu_\alpha(z), \quad f \in A^p_\alpha,\ g \in A^q_\alpha,$$

where $\frac{1}{p} + \frac{1}{q} = 1$. Let $\varphi \in S(\mathbb{D})$, the composition operator C_φ is defined as

$$C_\varphi f(z) = f(\varphi(z)), \quad f \in H(\mathbb{D}),\ z \in \mathbb{D}.$$

It is well known that C_φ is always bounded on A^p_α when $\varphi \in LFT(\mathbb{D})$, the set of linear fractional transformations of $\mathbb{D}$.

Let $L(X)$ denote the space of continuous linear operators on a separable, infinite dimensional Banach space X. A continuous linear operator $T \in L(X)$ is said to be *hypercyclic* if there is an $x \in X$ such that $orb(T, x) = \{T^n x : n \geq 0\}$ is dense in X. In such a case, x is called a hypercyclic vector for T. Besides, a vector $x \in X$ is called *supercyclic* for T if its projective orbit, $\{\lambda T^n x; n \geq 0, \lambda \in \mathbb{C}\}$ is dense in X. A continuous linear operator T acting on a separable Banach space X is said to be *mixing*, if for any pair U, V of nonempty open subsets of X, there exists some $N \geq 0$ such that $T^n(U) \cap V \neq \emptyset$ for all $n \geq N$.

The hypercyclicity of composition operators in one complex variable has been discussed in [8,12,15]. For composition operators with linear fractional symbols on $H^2(\mathbb{B}_N)$, some results have been obtained in [9,13]. Besides, Gallardo-Gutiérrez and Montes-Rodríguez [10] characterized the different cyclic properties of scalar multiples of linear fractional composition operators on weighted Dirichlet spaces.

The lower density of a subset A of $\mathbb{N}$ is defined as

$$\underline{dens}(A) = \liminf_{N\to\infty} \frac{card\{0 \leq n \leq N; n \in A\}}{N+1}.$$

A vector $x \in X$ is called *frequently hypercyclic* for T, if for every non-empty open subset U of X,

$$\underline{dens}\{n \in \mathbb{N} : T^n x \in U\} > 0.$$

It is obvious that if the operator T is frequently hypercyclic, then T is hypercyclic.

The following statement, that is due to Bayart and Grivaux [1], furnishes a sufficient condition for frequent hypercyclicity.

Theorem 1.1 (*Frequent Hypercyclicity Criterion*)**.** *Let T be an operator on a separable Banach space X. If there is a dense subset X_0 of X and a map $S : X_0 \to X_0$ such that, for any $x \in X_0$,*