Available online at www.sciencedirect.com

SciVerse ScienceDirect

Indagationes Mathematicae 24 (2013) 602–612

indagationes mathematicae

www.elsevier.com/locate/indag

A blow-up result in a system of nonlinear viscoelastic wave equations with arbitrary positive initial energy

Mohammad Kafini*, Salim A. Messaoudi

Department of Mathematics and Statistics, KFUPM, Dhahran 31261, Saudi Arabia

Received 14 January 2013; accepted 3 April 2013

Communicated by G.J.B. van den Berg

Abstract

In this paper we consider a system of nonlinear viscoelastic wave equations. Under arbitrary positive initial energy and standard conditions on the relaxation functions, we prove a finite-time blow-up result.

Keywords: Blowup; Nonlinear wave; Viscoelastic; Finite time

1. Introduction

In this work we consider a system of nonlinear viscoelastic wave equations in the presence of linear damping and the nonlinear source, namely,

$$\begin{cases} u_{tt} - \Delta u + \int_0^t g(t-s)\Delta u(s)ds + \mu_1 u_t = f_1(u,v), & x \in \Omega,\ t > 0, \\ v_{tt} - \Delta v + \int_0^t h(t-s)\Delta v(s)ds + \mu_2 v_t = f_2(u,v), & x \in \Omega,\ t > 0, \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), & x \in \Omega, \\ v(x,0) = v_0(x), \quad v_t(x,0) = v_1(x), & x \in \Omega, \\ u\,|_{\partial\Omega} = v\,|_{\partial\Omega} = 0, \end{cases} \tag{1.1}$$

* Corresponding author. Tel.: +966 502850766.
E-mail addresses: mkafini@dcc.kfupm.edu.sa, mkafini@kfupm.edu.sa (M. Kafini), messaoud@kfupm.edu.sa (S.A. Messaoudi).

0019-3577/$ - see front matter
http://dx.doi.org/10.1016/j.indag.2013.04.001

where $\Omega \subset \mathbb{R}^n$ is a bounded domain with a smooth boundary $\partial\Omega$. The functions g, h are the relaxation functions, nonnegative, nonincreasing and satisfying additional conditions to be specified later. This type of problems arise in viscoelasticity and in systems governing the longitudinal motion of a viscoelastic configuration obeying a nonlinear Boltzmann model. The functions u and v represent the transverse displacements of the waves.

The wave equation, in the absence of the viscoelastic term ($g = 0$), has been extensively studied and many results concerning global existence and nonexistence have been proved. For instance, for the equation

$$u_{tt} - \Delta u + a u_t |u_t|^m = b|u|^p u, \quad \text{in } \Omega \times (0, \infty) \tag{1.2}$$

$m, p \geq 0$, it is well known that, for $a = 0$, the source term $bu|u|^p$, $(p > 0)$ causes finite time blowup of solutions with negative initial energy (see [3]). The interaction between the damping and source terms was first considered by Levine [10,11] in the linear damping case ($m = 0$). He showed that solutions with negative initial energy blow up in finite time. Georgiev and Todorova [5], Messaoudi [17] and others extended Levine's result to the nonlinear damping case ($m > 0$).

In the presence of the viscoelastic term, Messaoudi [18] considered the following initial–boundary value problem

$$\begin{cases} u_{tt} - \Delta u + \int_0^t g(t-\tau)\Delta u(\tau)d\tau + u_t|u_t|^{m-2} = u|u|^{p-2}, & \text{in } \Omega \times (0,\infty) \\ u(x,t) = 0, \quad x \in \partial\Omega, t \geq 0 \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \Omega, \end{cases} \tag{1.3}$$

and showed, under suitable conditions on g, that solutions with initial negative energy blow up in finite time if $p > m$ and continue to exist if $m \geq p$.

Concerning Cauchy problems, Kafini and Messaoudi [8] established a blow-up result for the problem

$$\begin{cases} u_{tt} - \Delta u + \int_0^t g(t-s)\Delta u(x,s)ds + u_t = |u|^{p-1}u, & x \in \mathbb{R}^n, \ t > 0 \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \mathbb{R}^n \end{cases} \tag{1.4}$$

where g satisfies $\int_0^t g(s)ds < (2p-2)/(2p-1)$ and the initial data are compactly supported with negative energy such that $\int_{\mathbb{R}^n} u_0 u_1 dx \geq 0$. In their work, they extended the result of [22] established for the wave equation.

Recently, Maxim Korpusov [9] studied the initial–boundary-value problem for the generalized dissipative high-order equation of Klein–Gordon type with arbitrary positive initial energy. He established a blow-up result using the modified concavity method of Levine developed in [2]. More and recent blow-up results with arbitrary positive initial energy can be found, for instance, in Jie Ma et al. [15], Hongrui Geng et al. [4], Shengjia Lia et al. [12], and Shun-Tang Wu [21].

For a coupled system, Agre and Rammaha [1] investigated the following system

$$\begin{cases} u_{tt} - \Delta u + u_t|u_t|^{m-1} = f_1(u,v), & \text{in } \Omega \times (0,T) \\ v_{tt} - \Delta v + v_t|v_t|^{r-1} = f_2(u,v), & \text{in } \Omega \times (0,T) \end{cases} \tag{1.5}$$

where $\Omega \subseteq \mathbb{R}^n$ ($n = 1, 2, 3$) is a bounded domain with smooth boundary and $m, r \geq 1$. Under special types of the nonlinearities f_1 and f_2, they established the global existence and blowup of weak solutions with initial and Dirichlet boundary conditions.