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ScienceDirect
Indagationes Mathematicae 24 (2013) 939–970
indagationes mathematicae
www.elsevier.com/locate/indag

The combinatorics of N.G. de Bruijn

Ton Kloks[a], Rob Tijdeman[b,*]

[a] Department of Computer Science, National Tsing Hua University, Taiwan
[b] Mathematical Institute, Leiden University, The Netherlands

Abstract

In memoriam: N.G. de Bruijn.

In this article we present a survey of his papers on combinatorics. The section titles show its variety.

1. Common systems of representatives
2. De Bruijn cycles
3. The De Bruijn–Erdös theorem from incidence geometry
4. Bases for integers
5. The BEST theorem
6. The De Bruijn–Erdös theorem from graph theory
7. Factorizations of finite groups
8. Rooted trees in the plane
9. Permutations of a given shape
10. Covering of graphs by dimers
11. Counting (Pólya's fundamental theorem, Color designs)
12. Penrose tilings

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Keywords: Combinatorics; De Bruijn–Erdös theorem; Permutations; Counting; Penrose tilings

1. Common systems of representatives

In 1927, Van der Waerden published his theorem on a common system of representatives of two partitions of a finite set. To be precise, his theorem reads as follows [101].

* Corresponding author. Tel.: +31 715893046; fax: +31 715277101.
E-mail addresses: kloks@cs.nthu.edu.tw (T. Kloks), tijdeman@math.leidenuniv.nl (R. Tijdeman).

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doi:10.1016/j.indag.2012.12.001

Theorem 1. *Let M be a finite set. Let $\mathcal{U}$ and $\mathcal{B}$ be two partitions*[1] *of M such that every element of $\mathcal{U}$ and every element of $\mathcal{B}$ have exactly n elements. Let*

$$\mu = |\mathcal{U}| = |\mathcal{B}|.$$

Then there is a set $X \subset M$ of μ elements such that for every $x \in X$ there are precisely one $U \in \mathcal{U}$ and precisely one $B \in \mathcal{B}$ such that

$$X \cap U = X \cap B = \{x\}.$$

In other words, X is a common system of representatives for $\mathcal{U}$ and $\mathcal{B}$.

In an afterword Van der Waerden explains that his theorem is equivalent with the theorem of König which says that every regular bipartite graph has a cover with dimers (see Section 10).[2] Van der Waerden mentions that a paper of König and Valkó [68] extends König's theorem to hold for infinite, regular bipartite graphs of finite degree. This extension implies the extension of Van der Waerden's theorem to the infinite. Here, possibly $|M| = \infty$, but all elements of $\mathcal{U}$ and $\mathcal{B}$ have the same, finite number of elements.

Extensions to the infinite of the closely related Marriage Theorem of Hall were obtained by Rado in 1965 [82]. In the finite case Hall's theorem is equivalent with the theorem of König–Egerváry. This theorem says that, for bipartite graphs, the maximal number of edges such that no two edges have a vertex in common equals the minimal cardinality of a set of vertices such that every edge has an endpoint in the set. This implies, of course, König's theorem which we mentioned above.

Rado mentions also earlier versions of the extension of Hall's theorem, viz by Hall himself in 1948 [59] and by others [51,60]. These early extensions (by Hall, by Everett and Whaples, and by Halmos and Vaughan) make use of Tychonoff's theorem (see Section 6). Rado's proof does not use Tychonoff's theorem. Moreover, Rado proves much more than just the extension which we mention below in Theorem 2.

Theorem 2. *Let N be a finite or infinite set and for every $k \in N$ let A_k be a finite set. Suppose that Hall's condition is satisfied, that is, for every finite $M \subset N$ we have*

$$\left| \bigcup_{k \in M} A_k \right| \geqslant |M|.$$

Then there is an $x_k \in A_k$, for every $k \in \mathbb{N}$, such that

$$k \neq \ell \quad \textit{implies} \quad x_k \neq x_\ell.$$

Prior to all this, in 1943, a paper of De Bruijn [15] appears that contains another extension of Van der Waerden's theorem. In De Bruijn's version M may be an infinite set. The requirement that all elements of $\mathcal{U}$ and of $\mathcal{B}$ contain the same, finite number of elements is replaced by the following two conditions.

[1] A partition of a set V is a collection of nonempty disjoint subsets of V, the union of which is V.
[2] A 'cover with dimers' is nowadays better known as a 'perfect matching'.

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