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Ideals with bounded approximate identities in the Fourier algebras on homogeneous spaces

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Abstract

In this paper, the closed ideals with bounded approximate identities in the Fourier algebra of an amenable homogeneous space is characterized in terms of the coset ring of the corresponding group. Some of these results are also extended to the case of Figà–Talamanca–Herz algebra. The closure of the Fourier algebra $A(G/K)$ in the cb-multiplier norm is also considered and we prove some results on spectral synthesis. We also derive some results on the ideals with bounded approximate identity.

Keywords: Homogeneous space; Fourier algebra; Figà–Talamanca–Herz algebra; Completely bounded multiplier; p-operator space; Spectral synthesis; Coset ring; Approximate identity; Amenability

1. Introduction

Let G be a locally compact group and let $A(G)$ denote the Fourier algebra $A(G)$ of G, introduced by Eymard [7]. One of the long standing open problems concerning the Fourier algebra $A(G)$ is the characterization of closed ideals with bounded approximate identity. Recently, Forrest et al. [14], have solved this problem in the case of an amenable locally compact group. This characterization is in terms of the elements of the coset ring of G.

Let G be a locally compact group containing a compact subgroup K. Let $A(G/K)$ denote the Fourier algebra corresponding to the homogeneous space G/K, introduced by Forrest [9]. In Section 3 of this paper, the ideals with bounded approximate identity in the Fourier algebra of

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an amenable homogeneous space are characterized. This characterization is also in terms of the elements of the coset ring of G. As a consequence, we obtain the relation between existence of bounded approximate identity in the closed ideal of $A(G/K)$ and the corresponding closed ideal of $A(G)$.

One of the main ingredients in the proof of Theorem 2.3 of [14] was the Host's generalization of Cohen's idempotent theorem to nonabelian groups. Theorem 3.5 of this paper gives the complete characterization of the idempotents in $B(G/K)$.

In [27], Ruan proved that for any locally compact group G, the Fourier algebra $A(G)$ is operator amenable if and only if G is amenable, which is the analogue of the Johnson's result on the amenability of $L^1(G)$. In [16], Forrest et al., showed that the corresponding statement for the Fourier algebra $A(G/K)$ fails. Also, they characterized the operator amenability of $A(G/K)$ for a class of locally compact groups called $[MAP]_K$ groups, in terms of the elements of the coset ring. Corollary 3.9 of this paper gives a complete characterization of when $A(G/K)$ is operator amenable.

In [10], Forrest defined and studied the algebra $A_0(G)$, the closure of the Fourier algebra with the cb-multiplier norm. He continued his study on this algebra in [11]. He showed that amenable subgroups are sets of spectral synthesis for $A_0(G)$. In Section 4 of this paper, we also define the algebra $A_0(G/K)$ for a homogeneous space G/K and derive some of its functorial properties. Also some results in connection with the ideals with bounded approximate identity are derived.

If H is a closed subgroup of G and if E is a closed subset of H, then E is a set of synthesis for $A(H)$ if and only if it is a set of synthesis for $A(G)$. This theorem is known as the injection theorem for sets of synthesis. In the abelian case this is a classical result of Reiter (see [26]). The nonabelian version is due to Kaniuth and Lau [22] and Parthasarathy and Prakash [24]. For the corresponding results on homogeneous spaces see [20,25]. (ii) and (v) of Theorem 4.6 of this paper presents the subgroup lemma and the injection theorem respectively for $A_0(G/K)$.

Finally, in Section 4, we extend some of the results of Section 3 to the case of Figà–Talamanca–Herz algebras. As a consequence we are able to derive a sufficient condition for the p-operator amenability of $A^p(G/K)$. We also generalize Theorem 2.2 of [28] to our case and use it to derive a necessary condition for the p-operator amenability of $A^p(G)$.

We begin with some of the required preliminaries in the next section.

2. Preliminaries

Let G be a locally compact group. Let π be a unitary representation of G on a Hilbert space $\mathcal{H}_\pi$. For $u, v \in \mathcal{H}_\pi$, let $\pi_{u,v}$ denote the coefficient function corresponding to π, u and v. The *Fourier–Stieltjes algebra* of G, denoted by $B(G)$, is defined as the collection of all coefficient functions arising from all the unitary representations. In [7], Eymard introduced the algebra $B(G)$. He showed that it is also the dual of the group C^*-algebra $C^*(G)$. With the dual norm $B(G)$ becomes a commutative Banach algebra with the pointwise addition and multiplication.

The closed linear span of all coefficient functions arising only from the left regular representation, ρ, is called as the *Fourier algebra* of the group G, denoted by $A(G)$. In [7], it is proved that $A(G)$ is also a commutative Banach algebra. Further, $A(G)$ is also a regular, semisimple Banach algebra with the Gelfand spectrum homeomorphic to G. It is also a closed ideal of $B(G)$. For more on the Fourier and the Fourier–Stieltjes algebra, we refer the readers to the fundamental paper of Eymard [7].

Let G be a locally compact group and K a compact subgroup of G. The Fourier algebra of the homogeneous space G/K, denoted by $A(G/K)$, was defined and studied by Forrest in [9].