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Existence results for abstract fractional differential equations with nonlocal conditions via resolvent operators

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Abstract

In our paper [10] we discussed an error in the recent literature on abstract fractional differential equations and we proposed a different approach to treat these type of problems. By noting that the results by Hernández et al. (2010) [10] are not applicable for problems with nonlocal conditions, in this paper we study the existence of mild solutions for a class of abstract fractional differential equations with nonlocal conditions. An application involving a partial differential equation with a fractional temporal derivative and nonlocal conditions is considered.

Keywords: Fractional derivatives; Abstract Cauchy problem; Resolvent of linear operators

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1. Introduction

In [10], we discussed an error in the recent literature on abstract fractional differential equations. By using an approach similar to that in [10], in this paper we study the existence of a mild solution for a class of abstract fractional differential equations with nonlocal conditions of the form

$$D^{\alpha}x(t) = Ax(t) + f(t, \mathcal{B}x(t), x(t)), \quad t \in [0, a], \tag{1.1}$$

$$x(0) = x_0 + g(x), \tag{1.2}$$

where $\alpha \in (0, 1)$, $A : D(A) \subset X \to X$ is the infinitesimal generator of a C_0-semigroup of bounded linear operators $(T(t))_{t\geq 0}$ defined on a Banach space $(X, \|\cdot\|)$, $x_0 \in X$, $f : [0, a] \times X^2 \to X$, $g : C(I; X) \to X$ are continuous functions, $\mathcal{B} : C([0, a]; X) \to C([0, a]; X)$ is given by $\mathcal{B}x(t) = \int_0^t B(t, s)x(s)ds$ and $\{B(t, s) : a \geq t \geq s \geq 0\}$ is a set of bounded linear operators on X such that $B(t, \cdot)x \in C([0, t]; X)$ and $B(\cdot, s)x \in C([s, a]; X)$ for all $t, s \in [0, a]$ and each $x \in X$.

The problem of the existence of mild solutions for abstract differential equations with a fractional temporal derivative has been considered in several recent papers; see [1,2,5,13,23, 22,25,27,30,32,33] and the papers [12,20,24,32,33] for problems with nonlocal conditions. As pointed out in [10], the results in these papers are incorrect since the considered concept of mild solution is not appropriate.

Motivated by the fact that the results in [10] are non applicable for problems with nonlocal conditions, in this paper we introduce a concept of mild solution for the problem (1.1)–(1.2) and we study the existence of mild solutions for (1.1)–(1.2). Concerning the above, we note that the results in [10] are related to the existence of local in time solutions and that the nature of the problem (1.1)–(1.2) requires the existence of solutions defined on the whole interval $[0, a]$. We also observe that the concept of mild solution in [10] is different to the concept introduced in Definition 2.1.

The general literature on abstract differential equations with nonlocal conditions is extensive and considers different topics on the existence and qualitative properties of solutions are considered. Concerning the motivations, relevant developments and the current status of the theory we refer the reader to [3,4,26,17,15,16], for the recent papers [7,11] and the references therein. On the other hand, the theory of fractional differential equations has received much attention over the past twenty years. Concerning the literature on fractional differential equations we cite the books [14,21,28], the recent papers [31,18,9,8,6] and the references therein.

We introduce now some definitions, properties and technicalities. Let $(Z, \|\cdot\|_Z)$ and $(W, \|\cdot\|_W)$ be Banach spaces. We denote by $\mathcal{L}(Z, W)$ the space of bounded linear operators from Z into W endowed with the operator norm denoted by $\|\cdot\|_{\mathcal{L}(Z,W)}$ and we write simply $\mathcal{L}(Z)$ and $\|\cdot\|_{\mathcal{L}(Z)}$ when $Z = W$. The notation $\mathcal{D}$ stands for the domain of the operator A provided with the graph norm $\|x\|_{\mathcal{D}} = \|x\| + \|Ax\|$. In addition, $B_r(x, Z)$ represents the closed ball with center at x and radius r in Z. As usual $C([0, a]; Z)$ denotes the space of all the continuous function from $[0, a]$ into Z with the sup-norm denoted by $\|\cdot\|_{C([0,a];Z)}$ and $C^{\gamma}(I; Z)$, $\gamma \in (0, 1)$, represents the space formed by all the Z-valued γ-Hölder continuous functions from $[0, a]$ into Z with the norm $\|u\|_{C^{\gamma}([0,a];Z)} = \|u\|_{C([0,a];Z)} + [|u|]_{C^{\gamma}([0,a];Z)}$ where $[|u|]_{C^{\gamma}([0,a];Z)} = \sup_{t,s\in[0,a],t\neq s} \frac{\|u(t)-u(s)\|_Z}{(t-s)^{\gamma}}$.