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Large deviations from a macroscopic scaling limit for particle systems with Kac interaction and random potential ☆

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Abstract

We consider a lattice gas in a periodic d-dimensional lattice of width γ^{-1}, $\gamma > 0$, interacting via a Kac's type interaction, with range $\frac{1}{\gamma}$ and strength γ^d, and under the influence of a random one body potential given by independent, bounded, random variables with translational invariant distribution. The system evolves through a conservative dynamics, i.e. particles jump to nearest neighbor empty sites, with rates satisfying detailed balance with respect to the equilibrium measures. In [M. Mourragui, E. Orlandi, E. Saada, Macroscopic evolution of particles systems with random field Kac interactions, Nonlinearity 16 (2003) 2123–2147] it has been shown that rescaling space as γ^{-1} and time as γ^{-2}, in the limit $\gamma \to 0$, for dimensions $d \geqslant 3$, the macroscopic density profile ρ satisfies, a.s. with respect to the random field, a non-linear integral partial differential equation, having the diffusion matrix determined by the statistical properties of the external random field. Here we show an almost sure (with respect to the random field) large deviations principle for the empirical measures of such a process. The rate function, which depends on the statistical properties of the external random field, is lower semicontinuous and has compact level sets.

Résumé

On considère un modèle de spins évoluant dans le tore de dimension $d \geqslant 3$, de largeur γ^{-1} ($\gamma > 0$), soumis à un potentiel d'interaction de Kac de portée γ^{-1} et à un champ extérieur aléatoire. Le champ extérieur aléatoire est défini par des variables aléatoires indépendantes, bornées dont la loi est supposée invariante par translation. L'évolution du système au cours du temps consiste à échanger l'occupation entre deux sites voisins selon des taux vérifiant la condition du bilan détaillé. La limite hydrodynamique a été étudiée en dimension $d \geqslant 3$ dans [M. Mourragui, E. Orlandi, E. Saada, Macroscopic evolution of particles systems with random field Kac interactions, Nonlinearity 16 (2003) 2123–2147]. Les auteurs ont démontré que sous l'échelle spatiale γ^{-1} et l'échelle temporelle γ^{-2}, pour presque tout environnement aléatoire, les mesures empiriques convergent vers l'unique solution faible d'une équation de second ordre définie à partir d'une matrice de diffusion. Dans ce papier nous établissons pour presque tout environnement aléatoire, un principe de grandes déviations pour ce modèle. La fonctionnelle d'action associée aux grandes déviations est semi-continue inférieurement et admet des ensembles de niveaux compacts.

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1. Introduction

Models where a stochastic contribution is added to the energy of the system naturally arise in condensed matter physics where the presence of the impurities causes the microscopic structure to vary from point to point. An extensive literature has been dedicated to study the equilibrium statistical properties of (spin) systems with external random field. The central question heatedly discussed in the 1980's in the physics community was whether the Random Field Ising model would show spontaneous magnetization at low temperature and weak disorder in dimension 3, or not. The problem was solved by Bricmont and Kupianen, [4], who proved the existence of phase transition in $d \geqslant 3$ for small magnitude of the random field, and Aizenman and Wehr, [1], who proved that there is no phase transition in $d = 2$ for all temperatures. A more physical oriented review about Random Field Ising model is [22].

The Kac's potentials are two body interactions with range $\frac{1}{\gamma}$ and strength γ^d, where $\gamma > 0$ is a dimensionless parameter which represents the ratio between microscopic and macroscopic lengths. When $\gamma \to 0$, i.e. very long range compared with the interparticle spacing, the strength of the interaction becomes very weak, but in such a way that the total interaction between one particle and all the others is kept finite. They were introduced in [12], and then generalized in [18], to present a rigorous derivation of the van der Waals theory of a gas–liquid phase transition. In the last decade many authors studied the equilibrium statistical properties of systems with Kac potential for γ small but finite and the time evolution of the macroscopic density profile in particle systems interacting via long range Kac potential either in the case of conservative dynamics [17,9,10,20], or in the case of non-conservative dynamics [7]. For a review of various results concerning these models, see [11,23,3]. Random Field Kac models, in $d = 1$ and for γ small and fixed, have been recently studied in [5,6]. The particle model studied in [21] and here is a dynamic version of lattice gases interacting via a two-body Kac interaction and subject to external random field given by independent bounded random variables with translational invariant distribution. The formal Hamiltonian we consider is given by

$$H_\gamma^{\beta,\alpha}(\eta) = -\frac{\beta}{2} \sum_{x,y\in\mathbb{Z}^d} J_\gamma(x-y)\eta(x)\eta(y) - \sum_{x\in\mathbb{Z}^d} \alpha(x)\eta(x), \tag{1.1}$$

where β is a positive parameter and $\eta(x) \in \{0,1\}$, $\eta(x) = 1$ if there is a particle in x and $\eta(x) = 0$ means that the site is empty. The $\{\alpha(x), x \in \mathbb{Z}^d\}$ represents the external random field on the sites x. Given the Hamiltonian (1.1) there is a standard way, see for example [28,16], to construct a dynamic which conserves the number of particles and for which the invariant measures are given by the one parameter family of Gibbs measures associated to (1.1). More precise statements will be given in Section 2. The relevant features of the systems associated to (1.1) are the absence of translation invariance, for a given disorder configuration, and the non-validity of the so called gradient condition. To establish the hydrodynamic limit one needs to show some version of Fick's law, namely to replace the microscopic current (i.e. the difference between the rate at which a particle jumps from site x to site y and the rate at which a particle jumps from site y to site x, x and y being nearest neighbors) by the gradient of the density field multiplied by the diffusion coefficient. Roughly speaking, the gradient condition says that the microscopic current is already the gradient of a function of the density field. Performing a diffusive scaling limit, in [21], for almost all disorder, a law of large numbers when $d \geqslant 3$ was established for the density field, starting from a sequence of measures associated to some initial density profile ρ_0, $0 \leqslant \rho_0 \leqslant 1$. The equation obtained for the density field is the following non-local, non-linear partial differential equation

$$\frac{\partial \rho}{\partial t} = \frac{1}{2}\nabla \cdot \left(\sigma(\rho)\nabla \frac{\delta \mathcal{G}}{\delta \rho}\right), \qquad \rho(0,r) = \rho_0(r), \tag{1.2}$$

where the energy functional $\mathcal{G}(\rho)$ is a suitable non-linear integral functional, see (2.27) and $\frac{1}{2}\sigma(\rho)$ is the mobility, see (2.22),[1] or conductivity, of the system with only short range interaction, i.e. corresponding to $\beta \equiv 0$ in (1.1).

[1] In the physical literature one writes the mobility as $\sigma_1(\rho) = \frac{1}{2}\sigma(\rho)$. We assumed this convection in [21]. So the $\sigma(\rho)$ in [21] does correspond to $\frac{1}{2}$ of the quantity denoted here with the same symbol.