



Stability of transverse dunes against perturbations: A theoretical study using dune skeleton model

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ABSTRACT

The *dune skeleton model* is a reduced model to describe the formation process and dynamics of characteristic types of dunes emerging under unidirectional steady wind. Using this model, we study the dependency of the morphodynamics of transverse dunes on the initial random perturbations and the lateral field size. It was found that (i) an increase of the lateral field size destabilizes the transverse dune to cause deformation of a barchan, (ii) the initial random perturbations decay with time by the power function until a certain time; thereafter, the dune shapes change into three phases according to the amount of sand and sand diffusion coefficient, and (iii) the duration time, until the transverse dune is broken, increases exponentially with increasing the amount of sand and sand diffusion coefficient. Moreover, under the condition without the sand supply from windward ground, the destabilization of transverse dune in this model qualitatively corresponds to the subaqueous dunes in water tank experiments.

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1. Introduction

Erosion due to wind sculpts deserts on Earth and surfaces on Mars, Titan into sand dunes such as barchan, transverse, longitudinal, star-shaped, and dome-shaped (McKee, 1979; Cooke et al., 1993; Bourke and Goudie, 2009; Rubin and Hesp, 2009; Bridges et al., 2012). As the dominant factors dictating several dune shapes, the steadiness of wind direction and the amount of available sand in each dune field are known (Livingstone and Warren, 1996). For example, a unidirectional wind generates barchans of a crescent-shaped or transverse dunes extending perpendicular to the wind direction depending on the amount of available sand, whereas a bidirectional wind generates longitudinal dunes extending parallel to the sum of two wind directional vectors. Additionally, a multidirectional wind generates star dunes consisting of multiple crest lines extending from the top to several directions.

As one of remarkable subject in dune studies, the stability of transverse dunes has remained to be an open issue for both geomorphology and geophysics. In rescaled water tank experiments, an isolated transverse dune was shown unstable and deformed into a set of barchans as the end-stage (Hersen et al., 2002; Endo et al., 2005; Groh et al., 2008; Reffet et al., 2010). Also, computer models have reproduced the qualitative and quantitative morphodynamics similar to subaqueous dunes (Nishimori et al., 1993,

1998; Werner, 1995; Kroy et al., 2002; Durán et al., 2005; Zhang et al., 2010; Katsuki et al., 2011). Recently, theoretical approaches for the stability of transverse dunes have been conducted by the methodology considered the mass conservation of sand and the sand flows on dunes (Niiya et al., 2010, 2012; Parteli et al., 2011; Melo et al., 2012). Especially, Niiya et al. consider the dynamics of a dune as a combination of two-dimensional cross sections (hereafter, 2D-CSs) parallel to the wind direction and proposed the *dune skeleton model* (hereafter, *DS model*) consisting of coupled ordinary differential equations. This model is based on several assumptions used in the previous analytical model named “aeolian/aqueous barchan collision dynamical equations (ABCDE)” by Katsuki and Nishimori. ABCDE is able to describe the collision dynamics of two 3D barchans focusing on the central 2D-CS of barchans (Katsuki et al., 2005; Nishimori et al., 2009).

So far, the *DS model* has successfully reproduced three typical shapes of dunes, straight transverse dune, wavy transverse dune, and barchan, depending on the amount of available sand and wind strength (Niiya et al., 2010). Moreover, the *reduced DS model*, which is a further simplified *DS model* with a two-variable ordinary differential equation, enabled the elucidation of the mechanism of transition between different dune shapes using a bifurcation analysis (Niiya et al., 2012). However, in these previous studies, the initial condition was given as a sinusoidal curve with small amplitude and single-wavelength. Thus, the stability of dunes against random perturbations is yet to be identified. In this study, we investigate the effect of initial perturbations on the stability of transverse dunes, and the field size effect on the stability is also studied.

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2. Dune skeleton model

The *DS model* covers the formation processes of barchans and transverse dunes, both of which are generated under a unidirectional steady wind. This model is roughly based on three considerations. First, the dunes consist of 2D-CSs as mentioned in the previous section. Second, a lateral distance between neighboring 2D-CSs is set constant. Third, a combination of two forms of sand movement, intra-sand movement within each 2D-CS and inter-sand movement between neighboring 2D-CSs, is considered to govern the macroscopic morphodynamics of dunes.

3D barchans are isolated on hard ground in both wind and lateral directions, whereas 3D transverse dunes are not isolated because they extend in the lateral direction. However, in the wind direction, the laterally extending 3D transverse dunes are assumed to be separated by inter-dune hard ground. In addition, considering the observation that the 2D-CSs of barchans and transverse dunes very roughly show a scale-invariant triangular shape, we assume that the angles of their upwind and downwind slopes (θ and φ , respectively as shown in Fig. 1) are maintained constant through their migration, irrespective of their size. From these assumptions, the horizontal (i.e., wind directional) position and the height of each 2D-CS are uniquely determined if only the coordinate (x, h) of its crest is given. Moreover, the empirical geometrical constants A, B, and C are introduced as

$$A = \frac{\tan \theta \tan \varphi}{\tan \theta + \tan \varphi}, \quad B = \frac{\tan \varphi}{\tan \theta + \tan \varphi}, \quad C = \frac{\tan \theta}{\tan \theta + \tan \varphi},$$

where A, B, and C are set to 1/10, 4/5, and 1/5, respectively, reflecting the typical 2D-CS profiles of real barchans and transverse dunes.

As mentioned above, sand flow is classified into two forms: (a) the intra-2D-CS flow and (b) the inter-2D-CS flow. The intra-2D-CS flow along the upwind slope is uniquely determined if the over-crest sand flux q and the incoming sand flux from the windward ground f^{in} are given (Fig. 1). The over-crest sand flux q determines the erosion rate of the 2D-CS's upwind slope and the over-crest blown sand deposits along the downwind slope or directly escapes to the leeward inter-dune ground. The deposition ratio T_E of the over-crest sand along the downwind slope is assumed as an increasing function of height of the specific form

$$T_E(h) = \frac{h}{1.0 + h}. \quad (1)$$

Note that $T_E(h)$ is termed as the *sand trapping efficiency* (Momiji and Warren, 2000) and Eq. (1) roughly represents the case of typical shear velocity $u_* = 0.4 \text{ m s}^{-1}$ at the dune crest, where h is the height in a meter unit. The quantity of q reflects the over-dune wind strength; here, we assume q , where $0.1 \leq q \leq 1.0$, to be constant in each desert field, independent of the 2D-CS's height. In addition, all of the incoming sand flux from the windward ground, f^{in} , deposits on the upwind slope of dunes.

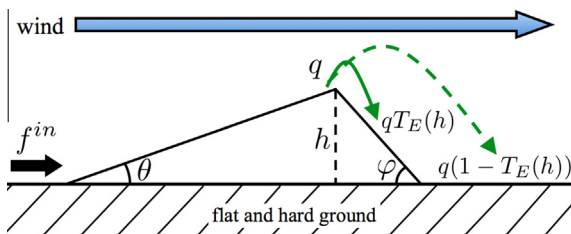


Fig. 1. Intra-2D-CS sand flow. The over-crest sand flux q and sand trapping efficiency T_E govern the intra-2D-CS flow. The green solid and dashed lines indicate sand deposition on the downwind slope and escaping sand to the leeward ground, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The inter-2D-CS flow $J_{u(i \rightarrow j)}/J_{d(j \rightarrow i)}$ occurs only between the upwind/downwind slopes of the neighboring 2D-CSs, i and j (Fig. 2). Locally, most of the lateral sand transport is determined by the height difference between neighboring 2D-CS's slopes. Therefore, we assume that the total flux is the sum of local sand transport from the slope's foot to the 2D-CS's crest. Namely, the flux is roughly considered as the lateral diffusion depending on the height difference, though the consideration of the overlap length of slopes causes a nonlinearity in the present form of the inter-2D-CS flux. The specific forms of $J_{u(i \rightarrow j)}$ and $J_{d(j \rightarrow i)}$ are

$$J_{u(i \rightarrow j)} = \begin{cases} \frac{D_u B}{2A\Delta w^2} \left\{ h_j^2 - \left[h_j - \frac{A}{B}(x_j - x_i) \right]^2 \right\} & x_j - x_i > 0 \\ \frac{D_u B}{2A\Delta w^2} \left\{ \left[h_i + \frac{A}{B}(x_j - x_i) \right]^2 - h_j^2 \right\} & x_j - x_i \leq 0, \end{cases} \quad (2a)$$

$$J_{d(j \rightarrow i)} = \begin{cases} \frac{D_d C}{2A\Delta w^2} \left\{ h_j^2 - \left[h_i - \frac{A}{C}(x_j - x_i) \right]^2 \right\} & x_j - x_i > 0 \\ \frac{D_d C}{2A\Delta w^2} \left\{ \left[h_j + \frac{A}{C}(x_j - x_i) \right]^2 - h_i^2 \right\} & x_j - x_i \leq 0, \end{cases} \quad (2b)$$

where Δw , set as $\Delta w = 1$ hereafter, is the lateral interval between neighboring 2D-CSs. These quantities correspond to the colored areas in Fig. 2 multiplied by diffusion coefficients. Here, the upwind and downwind diffusion coefficients (D_u and D_d , respectively) control the amount of inter-2D-CS sand flow on the respective sides of the slopes and reflect the over-dune wind strength.

With consideration of the above intra- and inter-sand flows, the dynamics of the coordinates $(x_i, h_i) (i = 1, \dots, N)$ of the 2D-CS's crest are given as a system of coupled ordinary differential equations (Niiya et al., 2010, 2012):

$$\frac{dx_i}{dt} = \frac{1}{h_i} \left[q(BT_E(h_i) + C) + \sum_{j=i\pm 1} (BJ_{d(j \rightarrow i)} + CJ_{u(i \rightarrow j)}) - C_f^{in} \right], \quad (3a)$$

$$\frac{dh_i}{dt} = \frac{A}{h_i} \left[q(T_E(h_i) - 1) + \sum_{j=i\pm 1} (J_{d(j \rightarrow i)} - J_{u(i \rightarrow j)}) + f_i^{in} \right]. \quad (3b)$$

We also introduce the annihilation rule of the 2D-CSs at the lateral edges of a dune; this rule is required to simulate the shrinking process of dunes. This rule is applied in the cases where h_i is lower than 0 by the escaping of sand from i th 2D-CS (Fig. 3(a)) or where the overlap between the 2D-CS at the edge and its neighbor vanishes (Fig. 3(b)). If the annihilation rule is applied to i th 2D-CS, we consider that the state of i th lane gets corresponding to the exposed state of ground, that is, i th 2D-CS is taken out of the calculation though the virtual crest of height zero is fixed at the foot of downwind slope as

$$h_i = 0, \quad x_i = \max_{j=i\pm 1} \left[x_j + \frac{C}{A} h_j \right].$$

In order to compare the results obtained from *DS model* with the real desert dunes, we estimate the value of q , D_u , and D_d in physical units on the basis of the observed data in real dune fields. The migration velocity of single 2D-CS in our model is assumed as

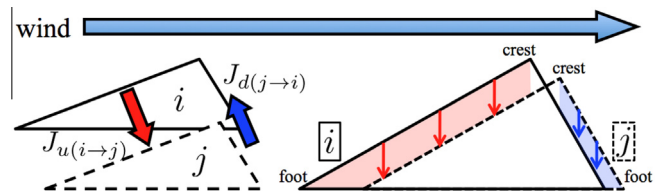


Fig. 2. Inter-2D-CS sand flow J_u (upwind) and J_d (downwind). Both the red and blue arrows indicate local sand transport depending on the height difference. The red and blue areas correspond to J_u and J_d , respectively, by summing the local sand transport from the slope's foot to the 2D-CS's crest. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

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