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ORIGINAL ARTICLE

Generalized covering approximation space and near concepts with some applications

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Abstract In this paper, we shall integrate some ideas in terms of concepts in topology. First, we introduce some new concepts of rough membership relations and functions in the generalized covering approximation space. Second, we introduce some topological applications namely "near concepts" in the generalized covering approximation space. Accordingly, several types of fuzzy sets are constructed. The basic notions of near approximations are introduced and sufficiently illustrated. Near concepts are provided to be easy tools to classify the sets and to help for measuring exactness and roughness of sets. Many proved results, examples and counter examples are provided. Finally, we give two practical examples to illustrate our approaches.

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1. Introduction

Rough set theory, a mathematical tool to deal with inexact or uncertain knowledge in information systems, has originally described the indiscernibility of elements by equivalence relations. Covering rough sets [1–9,11,12] is a natural extension of classical rough sets by relaxing the partitions arising from equivalence relations to coverings. In our work [6], we have introduced a framework to generalize covering approximation space that was introduced by Zhu [11]. In fact, we have introduced the generalized covering approximation space $\mathcal{G}_n - \boldsymbol{CAS}$ as a generalization to rough set theory and covering approximation space. The $\mathcal{G}_n - \boldsymbol{CAS}$ is defined by the triple $\langle U, \mathcal{R}, \mathcal{C}_n \rangle$, where $U \neq \emptyset$ be a finite set, $\mathcal{R}$ be a binary relation on U and $\mathcal{C}_n$ be n-cover of U associated to $\mathcal{R}$, where $n \in \{r, l\}$ (*for more details see* [6]).

The main works in this paper are divided into three parts. In the beginning of work, we introduce some new generalized definitions to rough membership relations (resp. functions) and new types of fuzzy sets in $\mathcal{G}_n - \boldsymbol{CAS}$. Second part aims to introduce one of an important topological concepts which are called "near concepts" in rough context (specially, in $\mathcal{G}_n - \boldsymbol{CAS}$). In fact, we apply near concepts in $\mathcal{G}_n - \boldsymbol{CAS}$ to define different tools for modifying the original operations. The suggested methods in this paper represent easy mathematical tools to approximate the rough sets and removing the uncertainty (vagueness) of sets. In addition, comparisons between the suggested methods are obtained and many examples (resp. counter examples) to illustrate these connections are provided. Hence, we can say that our approaches are very useful in rough context namely, in information analysis and in decision making. Finally, in the end of paper, simple practical examples are provided to illustrate the suggested methods and to show the importance of these methods in rough context namely in information system and in multi-valued information system. In addition, we give some comparisons between our approaches and others approaches such as Pawlak and Lin approaches.

2. *j*-Rough membership relations, *j*-rough membership functions and *j*-fuzzy sets

The present section is devoted to introduce new definitions for rough membership relations and functions as easy tools to classify the sets and help for measuring exactness and roughness of sets. These rough membership functions allow us to define four different fuzzy sets in $\mathcal{G}_n - \boldsymbol{CAS}$. Moreover, the suggested rough membership relations (resp. functions) are more accurate than classical rough membership function that was given by Lin [10] and the other types.

Definition 2.1. Let $\langle U, \mathcal{R}, \mathcal{C}_n \rangle$ be a $\mathcal{G}_n - \boldsymbol{CAS}$, and $A \subseteq U$. Then we say that:

(i) x is "j-surely" belongs to A, written $x \underline{\in}_j A$, if $x \in \underline{\mathcal{R}_j}(A)$.
(ii) x is "j-possibly" belongs to X, written $x \bar{\in}_j A$, if $x \in \overline{\mathcal{R}_j}(A)$.