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Nonlinear harmonic generation by diurnal tides

Scott Wunsch *

The Johns Hopkins University, Applied Physics Laboratory, 11100 Johns Hopkins Road, Laurel, MD, USA

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ABSTRACT

Recent observations from the South China Sea have demonstrated that semi-diurnal tides sometimes generate a double-frequency harmonic. Similar harmonic generation has been found in laboratory experiments and numerical simulations of internal wave beams refracting into a pycnocline. Here, a weakly nonlinear theory of internal wave refraction is applied to oceanic internal tides in an idealized stratification profile. The steady state harmonic amplitude is calculated as a function of the tidal frequency and the pycnocline characteristics. The results indicate that harmonic generation by nonlinear refraction of semi-diurnal tides is consistent with the South China Sea observations.

1. Introduction

In the oceans, a broad spectrum of internal wave frequencies and wave numbers is observed (Garrett, 1979; Holloway, 1980), bounded by the Coriolis and buoyancy frequencies. Energy is injected at lower frequencies, and various nonlinear effects transfer energy to higher frequencies, establishing the observed spectrum. Low frequency, low mode (long wavelength) internal tides are an important energy source, with a global generation rate on the order of 1 TW (Munk and Wunsch, 1998). This energy may be important both for ocean mixing (Garrett, 2003; Garrett and Kunze, 2007) as well as the observed internal wave spectrum. Multiple mechanisms have been proposed to explain the dissipation of energy in the internal tides. Among these are scattering to smaller scale modes by topography (Johnston and Merrifield, 2003; Balmforth and Peacock, 2009), parametric subharmonic instability (MacKinnon and Winters, 2005), interaction with mesoscale flow structures (Rainville and Pinkel, 2006), and damping at critical slopes (Kunze and Llewellyn-Smith, 2004). Each of these mechanisms is likely important, but no individual cause appears to be responsible for the majority of the required dissipation, and additional unidentified causes might also contribute.

Recent observations in the South China Sea (Xie et al., 2008, 2013) indicate the presence of a double-frequency harmonic of the semi-diurnal internal tide. This harmonic mode is transient, appearing and disappearing from the observed currents over periods of several days, and is plausibly due to the nonlinear transfer of energy from the primary internal tide. The simulations of Johnston and Merrifield (2003) for tidal flow over topography also generated a double-frequency harmonic in cases in which the stratification was non-uniform. The mechanism for harmonic generation is not fully understood, and could plausibly contribute to the dissipation of the internal tides and the formation of the internal wave spectrum. Various nonlinear effects may be responsible for harmonic generation.

There has been much progress in understanding the nonlinear mechanisms by which internal waves transfer energy to harmonic modes. One prominent example is the excitation of double-frequency harmonics during internal wave beam

* Tel.: +1 240 228 1859.
E-mail address: scott.wunsch@jhuapl.edu

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reflection from a solid surface (Tabaei and Akylas, 2003; Tabaei et al., 2005; Peacock and Tabaei, 2005; Rodenborn et al., 2011). The nonlinear wave–wave interaction between different wave numbers in the overlapping incident and reflected beams generates the harmonic waves. A similar effect in beam reflection from a free surface or a mixed layer has also been found in numerical simulations (Zhou and Diamessis, 2013). Nonlinear effects in colliding internal wave beams generate harmonics with frequencies equal to the sum and difference of the incident frequencies, as has been shown theoretically (Tabaei et al., 2005), in the laboratory (Smith and Crockett, 2014), and in simulations of oceanic beams (Lamb, 2004). Sufficiently wide internal wave beams are subject to the nonlinear parametric subharmonic instability, leading to the formation of two subharmonic beams whose frequencies sum to that of the original wave frequency (Bourget et al., 2013; Gayen and Sarkar, 2013). Internal tides may also be subject to this instability (MacKinnon and Winters, 2005; Xie et al., 2008).

The nonlinear generation of double-frequency harmonics by an internal wave beam incident on a pycnocline beneath a mixed layer has been recently demonstrated both experimentally (Mercier et al., 2012; Wunsch and Brandt, 2012) and numerically (Gayen and Sarkar, 2013; Grisouard and Staquet, 2010; Grisouard et al., 2011; Dossmann et al., 2013; Diamessis et al., 2014; Wunsch et al., 2014). Wave–wave interactions between the incident and reflected beams is one possible cause (Tabaei et al., 2005), analogous to solid surface reflection. This mechanism is proposed in Xie et al. (2013) as a possible explanation for the observed South China Sea harmonics, even though Xie et al. (2013) states that no direct evidence of beams was seen in the data. Nonlinear refraction through rapidly increasing stratification has been proposed as an alternate explanation for harmonic generation at a pycnocline (Diamessis et al., 2014; Wunsch et al., 2014). Unlike the overlapping beam mechanism, nonlinear refraction does not require wave–wave interactions. This implies that, for internal tides, a single mode may produce a double-frequency harmonic without any wave–wave interactions. Hence nonlinear refraction might explain the harmonics observed in the South China Sea by Xie et al. (2008, 2013) without invoking the presence of beams. The numerical simulations of internal tides by Johnston and Merrifield (2003) only exhibited higher harmonics when variable stratification was used, consistent with this nonlinear refraction conjecture. By transferring energy into higher harmonics, nonlinear refraction might contribute to the global internal tide dissipation rate. In addition, it could help populate the spectrum of internal waves observed in the oceans.

Here, the weakly nonlinear theory of internal wave refraction developed in Diamessis et al. (2014) and Wunsch et al. (2014) is extended to low mode internal tides. The effect of rotation is neglected. Although rotation is important for internal tide dynamics in most instances, the approximation of zero rotation may be adequate for the comparisons made to Xie et al. (2008, 2013), which were made in the tropical ocean where the Coriolis frequency is significantly less than the tidal frequency. The resulting estimate for the steady state harmonic amplitude compares favorably to the observations of Xie et al. (2013). This demonstrates that nonlinear refraction could plausibly be the mechanism of double-frequency harmonic generation.

2. Weakly nonlinear theory

Using a theoretical framework for weakly nonlinear effects in a two dimensional Bousinesq fluid (Tabaei and Akylas, 2003; Tabaei et al., 2005), refraction in variable stratification has recently been shown to result in harmonic generation (Diamessis et al., 2014; Wunsch et al., 2014). The stream function $\psi(x, z, t)$ $(u \equiv \partial_z \psi, w \equiv -\partial_x \psi)$ is decomposed into a primary mode ψ_1, with frequency ω and horizontal wavenumber k, and a (smaller amplitude) harmonic mode ψ_2, with double the incident frequency and wavenumber, as

$$\psi(x, z, t) = \psi_1(z)e^{i(\mathrm{kx}-\omega t)} + \psi_2(z)e^{2i(\mathrm{kx}-\omega t)} + c.c. \tag{1}$$

where *c.c.* denotes complex conjugate (so that the solutions are real). Both modes are assumed to have constant amplitude (steady state). The primary mode stream function obeys the well-known vertical structure equation (Drazin et al., 1981)

$$\partial_z^2 \psi_1 + k^2 \left(\left(\frac{N}{\omega} \right)^2 - 1 \right) \psi_1 = 0 \tag{2}$$

$$N^2(z) = -\frac{g}{\rho_o} \frac{d\rho}{dz} \tag{3}$$

Eq. (2) neglects rotation, and hence is applicable only at latitudes where $\omega^2 \gg f^2$, where f is the Coriolis frequency (Apel, 1987). The double-frequency harmonic mode, to lowest order and again neglecting rotation, obeys (Diamessis et al., 2014), (Wunsch et al., 2014)

$$\partial_z^2 \psi_2 + (2k)^2 \left(\left(\frac{N}{2\omega} \right)^2 - 1 \right) \psi_2 = \frac{k^3}{\omega^3} \left(\partial_z N^2 \right) \psi_1^2 \tag{4}$$

In uniform stratification N, Eq. (4) reduces to Eq. (2) for an internal wave mode with frequency 2ω and wavenumber $2k$. The right-hand sides are the nonlinear source terms due to refraction of the primary internal wave mode ψ_1, which vanish (to all orders) for uniform stratification (Tabaei et al., 2005). Eq. (4) shows that internal wave refraction through any vertically-varying stratification profile (non-zero dN^2/dz) generates a double-frequency harmonic. In general, numerical methods are needed to solve Eq. (4) for an arbitrary stratification profile.