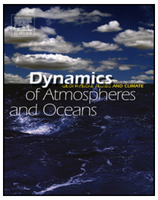
Dynamics of Atmospheres and Oceans 71 (2015) 98–107

Contents lists available at ScienceDirect

Dynamics of Atmospheres and Oceans

journal homepage: www.elsevier.com/locate/dynatmoce



On the instability of a buoyancy-driven downflow

Albert Dai [*]

Department of Engineering Science and Ocean Engineering, National Taiwan University, Taipei, Taiwan

ARTICLE INFO

Article history:
Received 11 September 2014
Received in revised form 30 June 2015
Accepted 30 June 2015
Available online 4 July 2015

Keywords:
Gravity currents
Buoyancy-driven flows
Instability

ABSTRACT

Gravity currents flowing downslope, namely downflows, were observed to have a larger scale instability on high slope angles and such violent instability was absent for downflows on low slope angles. By linear theory, it is found that two branches of instability occur for slope angle in the range of $0^\circ < \theta < 90^\circ$. The ensuing instability is on the upper branch for low slope angles and on the lower branch for high slope angles. There also exists a transitional slope angle, $\theta_E \approx 0.04^\circ$, at which the onset instability switches from one branch to the other. The scale of instability is found to increase and tend to skew towards the upper edge of the downflow as the ensuing instability switches from the upper branch to the lower one. Our findings surprisingly resonate with previously reported observations. Critical Reynolds number, below which the flow is stable to infinitesimal disturbances, is found to increase as the slope angle decreases. The role played by the bottom slope is essentially twofold. On one hand, the downslope component of gravity acts as the driving force for downflows. On the other hand, the wall-normal component of gravity acts for the stratification effect. Therefore, as the slope angle decreases, the driving force diminishes and the stratification intensifies, which can explain that the critical Reynolds number increases as the slope angle decreases. When a downflow propagates onto a sufficiently low slope angle, the low driving force and intensified stratification effect would make the downflow less prone to sustain a turbulent state of flow, which ultimately leads to the final stage of a gravity current event.

1. Introduction

Gravity currents, otherwise known as buoyancy or density currents, are flows of dense fluid beneath light ambient fluid, or light fluid above dense ambient fluid, primarily in the horizontal direction (Huppert, 2006). These flows are gravitationally driven by the density difference between the fluid in the currents and its environment and are quite common in natural and man-made environments (Allen, 1985; Fannelop, 1994; Simpson, 1997). A number of factors can cause variations in the density of fluid, which include temperature differentials, dissolved materials, and suspended sediments. While lock-exchange flows, in which gravity currents are produced from a finite buoyancy source released instantaneously on a horizontal boundary, have drawn much attention in the literature (e.g. Shin et al., 2004; Marino et al., 2005; Cantero et al., 2007; La Rocca et al., 2; Adduce et al., 2012), gravity currents over sloping terrain are also commonly encountered in nature. Prominent examples include dense overflows in the ocean (Nielsen et al., 2004; Dai and Garcia, 2010; Dai, 2010, 2013b) and powder-snow avalanches (Beghin et al., 1981; Hopfinger, 1983; Rastello and Hopfinger, 2004). In the laboratory, gravity currents on a slope can be produced from a sudden release of a finite volume of dense fluid (Maxworthy, 2010; Dai, 2013a) or

---

[*] Tel.: +886 2 33665790.
E-mail address: hdai@ntu.edu.tw

http://dx.doi.org/10.1016/j.dynatmoce.2015.06.004
0377-0265/

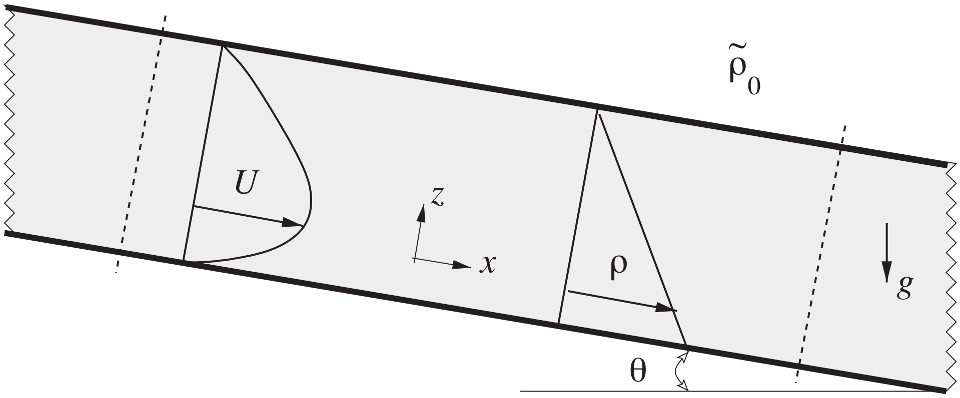


Fig. 1. Sketch of a downflow with a top wall driven by a linear density excess. The bottom slope makes an angle θ with the horizontal direction. The channel is open to the environment at both ends and there is no net pressure gradient acting on the flow. The flow represents an idealised model of a downflow in which entrainment of ambient fluid is not allowed in the channel.

from a steady source of dense fluid into a uniform (Britter and Linden, 1980) or stratified environment (Baines, 2001, 2005). Direct numerical simulations also provide a useful means to study the gravity currents on sloping boundaries (Cantero et al., 2009, 2009; Shringarpure et al., 2012). On a point of terminology, here we use the term 'downflow' to refer to the downslope propagating gravity current produced from a steadily maintained buoyancy inflow.

This paper is motivated by the recent observation reported by Talling et al. (2007) from the analysis of sediment cores that a transition of flow for a turbidity current event in the Agadir basin offshore from northwest Africa, marked by sediment deposition, could be triggered by a small but abrupt decrease of slope from 0.05° to 0.01°. The turbidity current propagated on the 0.05° slope for ∼100 km and continued on the 0.01° slope for a further ∼250 km. Sediment deposition was very little on the 0.05° slope and mainly on the 0.01° slope. This abrupt transition of flow is similar to the onset of hydrodynamic instability, but it was not determined unequivocally whether this flow transition is from one turbulent state to another or a reverse-transition from turbulent to laminar flows. Simple calculations and evidence for fast flow velocities as documented in Talling et al. (2007) indicate that the transition from one turbulent state to another is more likely, albeit without theoretical support.

Aiming at providing a theoretical support for the reported observation, we adopt a heuristic model, namely the 'downflow with a roof' configuration, and perform the stability analysis for the gravity currents down different bottom slopes. The analysis will be carried out based upon a two-dimensional buoyancy-driven base flow, of which the stability properties are of particular interest. Of course in the field the flow is rarely two-dimensional, however, from a theoretical point of view it is sufficient to consider only the two-dimensional problem to find the least stable conditions, i.e. Squire's theorem (Drazin and Reid, 1981). For a more thorough review and references on buoyancy-driven shear flow in a channel, the readers can be referred to Chen and Pearlstein (1989). In particular, it is worth noting that Birikh (1966), Birikh et al. (1968, 1972), Hart (1971) considered the stability of a plane-parallel flow in a channel where the top and bottom walls are heated differently. Such a stability problem is similar in configuration to but is different from the one investigated here. While in Birikh (1966), Birikh et al. (1968, 1972), Hart (1971) the buoyancy, or temperature, is maintained at different levels on the walls, i.e. Dirichlet type boundary conditions, here it is assumed no buoyancy flux across the boundaries, i.e. Neumann type boundary conditions. As will be discussed later, our results indicate two branches of instability which were not observed previously in these references.

The 'downflow with a roof' configuration has been widely employed in the study of buoyancy-driven flows on sloping boundaries, e.g. Cantero et al. (2009), Shringarpure et al. (2012). In this configuration, both ends of the channel are open to the environment and consequently, the flow within the channel is purely driven by the density excess of the downflow fluid. In reality, the downflows generally do not have a distinct upper boundary as shown in Fig. 1, but entrain ambient water across a more diffuse upper boundary. The thickness of the downflow, therefore, tend to slowly increase in the downslope direction due to ambient water entrainment. In the 'downflow with a roof' configuration, the slow development of the current as the downflow propagates is ignored by not allowing entrainment of ambient fluid. Notwithstanding, the 'downflow with a roof' configuration preserves a key element in the downflow problem, i.e. the flow is purely driven by the density excess, and allows for a steady state, uniform solution as a basis for the stability analysis.

To facilitate the analysis, we adopt a linear density excess profile, with a density excess $\Delta\tilde{\rho}_b$ at the bottom and no density excess on the top wall, within the channel for the body of a gravity current. That the density excess diminishes in the wall-normal direction away from the bottom can be a consequence of self-stratification effects in sediment-laden downflows, dilution due to entrained ambient fluid, or diffusion. There has been a large number of experimental works undertaken to analyse the velocity and density structures of a gravity current (McCaffrey et al., 2003; Choux et al., 2005; Felix et al., 2005). Simple as the linear density excess profile can be, it not only preserves the essential feature of the density profile in the body of a gravity current (Choux et al., 2005) but also self-consistently satisfies the equation of buoyancy