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# Nonlinear generation of harmonics through the interaction of an internal wave beam with a model oceanic pycnocline

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ABSTRACT

The interaction of an internal wave beam (IWB) with an idealized oceanic pycnocline is examined using two-dimensional fully nonlinear direct numerical simulations based on a spectral multidomain penalty method in the vertical direction. The phenomenon of focus is the nonlinear generation of harmonics. A total of 24 simulations have been performed, varying the normalized pycnocline thickness and the ratio of peak pycnocline Brunt-Väisälä frequency to that of the stratified lower layer. Harmonics at the point of IWB entry into the pycnocline increase in amplitude and number with a measure of the maximum gradient of the Brunt-Väisälä frequency, suggesting refraction as an important factor in harmonic generation. Among the simulations performed, two distinct limits of pycnocline thickness are identified. For thin pynoclines, whose thickness is 10% of the incident IWB's horizontal wavelength, harmonics trapped within the pycnocline have maximum amplitude when their frequency and wavenumber match those of the natural pycnocline interfacial wave mode. Results

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in this case are compared with weakly nonlinear theory for harmonic generation by plane wave refraction. For thicker pycnoclines, whose thickness is equal the incident IWB's horizontal wavelength, IWB refraction results in harmonic generation at multiple locations in addition to pycnocline entry, giving rise to complex flow structure inside the pycnocline.



## 1. Introduction

Internal wave beams (IWBs) are the high mode (small vertical scale) component of propagating internal wave motions radiated from isolated topographical features during the conversion of barotropic to baroclinic tide in the stratified ocean (Garrett, 2003; Kunze and Llewellyn-Smith, 2004; Garrett and Kunze, 2007). In contrast to the corresponding low mode (large vertical scale) component of the internal wave field, which transports energy away from the generation sites over distances as large as $O(1000\text{ km})$ (Ray and Mitchum, 1997; Zhao and Alford, 2009), IWB observations are restricted to the vicinity of the generating topography (Martin et al., 2006; Cole et al., 2009), as IWBs tend to dissipate much faster upon entry into the ocean pycnocline and soon thereafter.

Turbulent mixing and dissipation due to breaking of the IWB in the pycnocline have been suggested as possible causes of IWB degradation (Lien and Gregg, 2001; Althaus et al., 2003; Johnston et al., 2011) but other nonlinear interactions are also very likely operative. To this end, a number of recent observational studies (Althaus et al., 2003; Nash et al., 2006; Johnston et al., 2011) call for further investigation into the fate of an IWB incident on the pycnocline. Possible nonlinear interactions, alternative to wave breaking, include the "local" generation of internal solitary waves (ISWs) and generation of harmonic modes.

On one hand, the local generation of ISWs has been proposed as an alternative to more commonly reported generation mechanisms of oceanic ISWs, such as the interaction of the barotropic and baroclinic tide with the shelf-break (Maxworthy, 1979) and other isolated topographic features (Farmer and Armi, 1988, 1999; Stastna and Peltier, 2005) or the steepening of a Mode-I internal tide (Li and Farmer, 2011). The postulated formation of ISWs by an IWB incident on the ocean pycnocline has been observed in specific locations in the ocean, originally in the southern Bay of Biscay and off the Iberian peninsula (New and Pingree, 1990, 1992; New and Da Silva, 2002; da Silva et al., 2007) and more recently off the Mascarene Plateau near Mozambique (da Silva et al., 2009, 2011; New et al., 2013) and in the Red Sea (da Silva et al., 2012). These observations have served as the central motivation for a number of theoretical (Thorpe, 1998; Gerkema, 2001; Akylas et al., 2007; Mauge and Gerkema, 2008), laboratory (Mercier et al., 2012) and numerical studies (Grisouard and Staquet, 2010; Grisouard et al., 2011; Dossmann et al., 2013; Gayen and Sarkar, 2013) aimed toward further elucidating the underlying mechanism of local generation of ISWs.

On the other hand, the nonlinear generation of harmonic modes by incident IWBs on a pycnocline is a less explored phenomenon on which there appears to be only one, and very brief, reference in the observational oceanographic literature (Xie et al., 2013). Depending on environmental conditions, harmonic modes might plausibly propagate within the pycnocline, contribute to ISW formation, or re-radiate internal wave energy to the deep ocean at higher frequencies than the incident IWB. Occurring at smaller-wavelengths (Mercier et al., 2012; Wunsch and Brandt, 2012), harmonics may be responsible for both dissipation of IWB energy upon the beam's entry into the pycnocline and the transport of a significant fraction of this energy far from the IWB impact region, in the form of propagating pycnocline modes and, under certain conditions (Mercier et al., 2012), ISWs.

From a theoretical perspective, the work of Thorpe (1998) on the interaction of an incident plane internal wave with a thin pycnocline provides an analytical expression for the first harmonic wave at the base of the pycnocline. In Thorpe's analysis, the harmonics have double the incident frequency and wavenumber, and are strongest when these match a freely propagating interfacial wave on the pycnocline. Note that here, we will use the term "first" harmonic to refer to