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The statistical relation of sea-level and temperature revisited

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ABSTRACT

We propose a semi-empirical model for the relation between global mean surface temperature and global sea-levels. In contradistinction to earlier approaches to this problem, the model allows for valid statistical inference and joint estimation of trend components and interaction term of temperature and sea-level. Estimation of the model on the data set used in Rahmstorf (2007) yields a proportionality coefficient of 4.6 mm/year per °C at a one-sided significance level of 7.6 percent or higher. Long-term simulations of the model result in a two-sided 90-percent confidence interval for the sea-level rise in the year 2100 of [15 cm, 150 cm] above the 1990 level. This is a wider margin of error than was reported in the previous literature, and it reflects the substantial uncertainty in relating two trending time series.

1. Introduction

Estimating the influence of global temperature on global sea-level using time series of historical data is a statistical problem of observational data that has been discussed intensively in recent years

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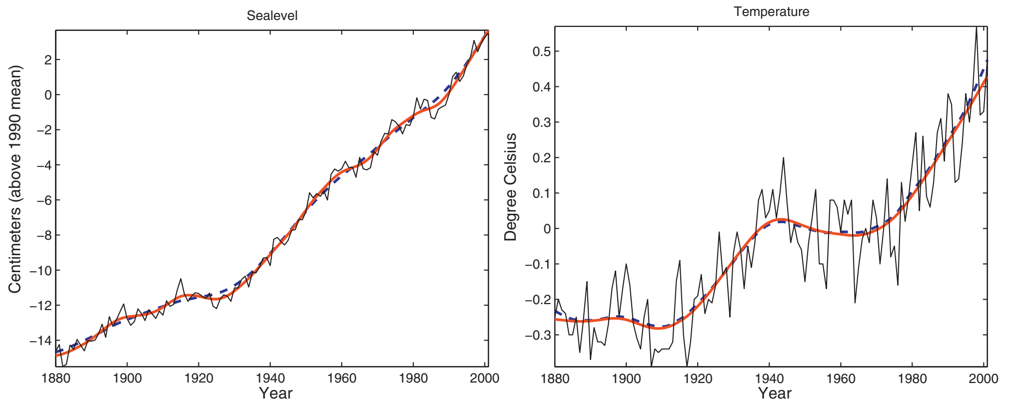

Fig. 1. Trend components in global sea-level (left) and global surface temperature (right). The solid smooth line is the trend component estimated from the state-space model. The dashed smooth line is the non-linear trend component estimated from singular spectrum analysis.

(Rahmstorf, 2007a,b, 2010; Woodworth et al., 2009; Horton et al., 2008, Jevrejeva et al., 2009, 2010). A central problem is the fact that both temperature and sea-level time series are trending upwards (see Fig. 1) and that the hypothesized relationship between the two lies exactly in these long-term trends. In other words, it is supposed that the long-term trend in global temperatures influences the long-term trend in global sea-levels (Rahmstorf, 2007; Rahmstorf and Vermeer, 2009). Therefore, standard statistical methods of differencing or de-trending and subsequent regression of stationary residuals onto each other are beside the point.

Semi-empirical models have been proposed that justify a linear approximation of the interaction term of these trend components in temperature and sea-level, which make the problem's estimation accessible by linear regression methods (Jevrejeva et al., 2009; Grinsted et al., 2010; Rahmstorf and Vermeer, 2009). In Rahmstorf (2007), Rahmstorf and Vermeer (2009), a linear approximate relationship for the influence of global temperatures on the differences of global sea-levels is motivated:

$$\frac{dH}{dt} = a(T - T_0), \tag{1}$$

where H is the sea-level, T is temperature, and a is the coefficient of influence. In order to estimate this relation, a linear regression of the type

$$f(H_t) - f(H_{t-1}) = b(f(T_t) - f(T_0)) + \text{error}_t \tag{2}$$

is run, where H_t is a time series of global sea-level observations, T_t is a time series of global temperature observations, b is the coefficient of influence in the model and an estimate of a in Eq. (1). For a given time series X_t, the data transformation $f(X_t)$ extracts a long-term trend. The extant literature often uses singular spectrum analysis in order to smooth short-run fluctuations, see for example Moore et al. (2005), Jevrejeva et al. (2006), Wahl et al. (2010), Rahmstorf et al. (2012a,b), Bittermann et al. (2013). Sometimes, moving averages over non-overlapping windows are applied in addition to singular spectrum analysis to further smooth the series (Rahmstorf, 2007; Rahmstorf and Vermeer, 2009). In the case of global sea-level and temperature time series H_t and T_t considered in Rahmstorf (2007), the regressand $f(H_t) - f(H_{t-1})$ and regressor $f(T_t) - f(T_0)$ are both upward trending.

The linear regression approach with trending time series potentially invalidates standard statistical inference (Schmith et al., 2007, 2012; Granger and Newbold, 1974). This is caused by the properties of the residual process (error_t), which may be statistically indistinguishable from a random walk. For the data set considered in this paper, we show that this is indeed the case (see Fig. 4 and Table 2). Smoothing by singular spectrum analysis adds to the complexity of the statistical properties of the process (error_t), leaving the asymptotic properties of the estimator of b in Eq. (2) intractable.