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Short communication

Note on the generalized thermal theory for gravity currents in the deceleration phase

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ABSTRACT

The generalized thermal theory for gravitational convection, produced from instantaneous buoyancy sources on sloping boundaries, developed in Dai and Garcia (2010) is examined in this note. An assumption implicitly made therein, that detrained fluid carries no momentum, was inappropriate and the solution was not physical in special cases. The generalized thermal theory is now improved by considering the momentum carried away by detrained mixed fluid. An asymptotic velocity–distance relation for gravity currents further downslope in the deceleration phase is provided and agreement with reported experimental data is found.

1. Introduction

Gravity currents, also known as density currents, are flows driven by density difference and are encountered in many man-made and natural environments. Gravity currents on slopes are commonly seen in situations such as powder-snow snow avalanches and turbidity currents off the continental shelf. Allen (1985), Fannelop (1994), and Simpson (1997) give a detailed account of the diversity of gravity currents and their relevance in natural sciences and engineering applications.

Much work reported so far in the literature has focused on gravity currents produced by an instantaneous, finite buoyancy source on a horizontal boundary, see for example Shin et al. (2004) for a review on lock-exchange flows. For gravity currents on a slope, the buoyancy source may be continuously maintained (Britter and Linden, 1980; Baines, 2001) or released instantaneously with finite buoyancy (Beghin et al., 1981; Dade et al., 1994; Rastello and Hopfinger, 2004). Concerning the terminology, gravitational convections of finite buoyancy released instantaneously are referred to interchangeably as the 'thermal cloud', 'boluse', or 'gravity currents' in the literature, and the term 'gravity currents' is chosen here.

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The thermal theory developed by Beghin et al. (1981) for instantaneously released gravity currents on sloping boundaries has formed the basis for many subsequent studies. The readers are referred to Maxworthy and Nokes (2007) and Dai and Garcia (2010) for a discussion of the model and its applications. Recently, the need to generalize the classic thermal theory has arisen, for example the buoyancy in the head during the acceleration phase can increase due to a following current (Maxworthy and Nokes, 2007) and the buoyancy in the head can decrease due to detrainment during the deceleration phase (Dai and Garcia, 2010).

Our focus is on the generalized thermal theory accounting for both entrainment and detrainment for gravity currents produced by an instantaneous buoyancy source on a slope. The aim in this note is to correct an arbitrary assumption implicitly made in Dai and Garcia (2010), to provide more evidence showing the influence of detrainment in the deceleration phase, and to generalize the thermal theory in a stably stratified environment (Appendix A).

2. Thermal theory revisited

2.1. Beghin, Hopfinger, and Britter's theory

Beghin et al. (1981) followed the famous Morton et al. (1956) and developed the thermal theory for gravitational convection produced from instantaneous bouyancy sources on sloping boundaries. The central assumption in thermal theory is the similarity hypothesis, which states that the form of the gravity current head, or 'thermal cloud', is semi-elliptical with constant aspect ratio during the motion on inclined surfaces in Beghin et al. (1981) and spherical for free vertical clouds in Morton et al. (1956). Based on the conservation of mass, momentum, and buoyancy, and with the assumption of entrainment, the analytical solutions for the height, length of the semi-elliptical head and the gravity current front velocity were presented (Fig. 1).

For the convenience of readers, the formulation of the original thermal theory is briefly summarized here. Note that the driving force for the gravity current is attributed to the original dense fluid contained within the 'thermal cloud'. Therefore, if the buoyancy were to be conserved in the cloud as assumed in the original thermal theory, the total driving force for the gravitational convection would be a constant. Here the ambient fluid density is taken as ρ_a and the density of original dense fluid is $\rho_a + \Delta\rho$, where $\Delta\rho/\rho_a \ll 1$. Following Beghin et al. (1981), the linear momentum with Boussinesq approximations takes the form

$$\frac{d\rho_a S_1 HLU}{dt} = B \sin\theta, \tag{1}$$

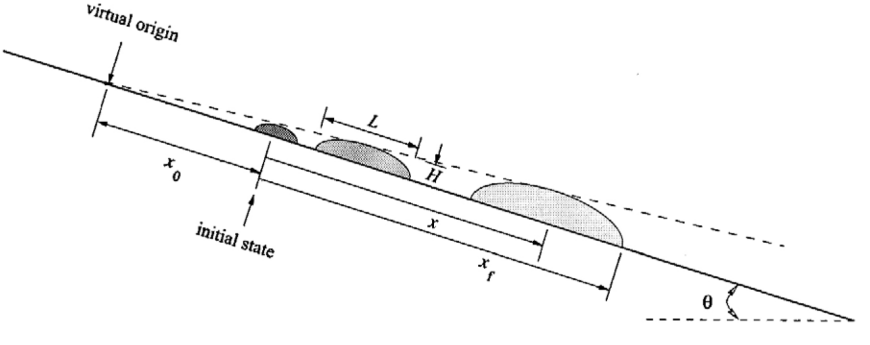

Fig. 1. Sketch for the gravity current produced from an instantaneous buoyancy source on a slope at an angle θ and the definitions in thermal theory. The gravity current travels in the downslope direction. H and L are the height and length of the current head; x_0 is the distance measured from the virtual origin to the initial state of motion; x is measured from the initial state to the mass-center of the semi-elliptical gravity current head; x_f is measured from the initial state to the current front. In the initial state, the cross-sectional area is A_0 and the mass-center velocity is U_0.