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Nonlinear adjustment of a localized layer of buoyant, uniform potential vorticity fluid against a vertical wall

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Abstract

The nonlinear evolution of a localized layer of buoyant, uniform potential vorticity fluid with depth H, width w_0 and length L released adjacent to a wall in a rotating system is studied using reduced-gravity shallow-water theory and numerical modeling. In the interior, far from the two ends of the layer, the initial adjustment gives, after ignoring inertia–gravity waves, a geostrophic flow of width w_∞ and layer velocities parallel to the wall directed in the downstream direction (defined by Kelvin wave propagation). This steady geostrophic flow serves as the initial condition for a semigeostrophic solution using the method of characteristics. At the downstream end, the theory shows that the fluid intrudes along the wall as rarefaction terminating at a nose of vanishing width and depth. However, in a real fluid the presence of the lower layer leads to a blunt gravity current head. The theory is amended by introducing a gravity current head condition that has a blunt bore joined to the rarefaction by a uniform gravity current. The upstream termination of the initial layer produces a Kelvin rarefaction that propagates downstream, decreasing the layer depth along the wall, and initiating upstream flow adjacent to the wall. The theoretical solution compares favorably to numerical solutions of the reduced-gravity shallow-water equations. The agreement between theory and numerical solutions occurs regardless of whether the numerical runs are initiated with an adjusted geostrophic solution or with the release of a stagnant layer. The latter case excites inertia–gravity waves that, despite their large amplitude and breaking, do not significantly affect the evolution of the geostrophic flow. At times beyond the validity of the semigeostrophic theory, the numerical solutions evolve into a stationary array of vortices. The vortex formation can be interpreted as the finite-amplitude manifestation of a linear instability of the new flow established by the passage of the Kelvin wave. The Kelvin wave ultimately reduces the flux into the downstream gravity current and the vortices retain buoyant in the neighborhood of the initial layer.

Keywords: Geostrophic adjustment; Gravity currents; Kelvin waves; Nonlinear waves; Vortices

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1. Introduction

In the classic geostrophic adjustment problem an initial geostrophically unbalanced state is allowed to relax to a final steady flow whose characteristics are determined by conserving certain properties of the initial state (e.g., mass, potential vorticity and angular momentum) (Rossby, 1938; Blumen, 1972). Often, the adjustment problem is considered for symmetric situations (e.g., the collapse of a cylindrical column) and removed from boundaries. If adjacent to boundary, the initial state is uniform in the direction parallel to the boundary. These restrictions render the adjustment process one dimensional, and in the case against a boundary, eliminate the possibility of Kelvin wave propagation along the boundary.

A notable exception is linear geostrophic adjustment in a channel considered by Gill (1976). In that problem a layer of fluid of depth h_1 is separated from a layer of the same fluid with depth $H > h_1$ by a dam that runs directly across the channel at $x = 0$. Removal of the dam excites a Kelvin wave that propagates downstream ($x > 0$) along the right-hand wall (looking from the deep to the shallow layer with anti-clockwise rotation). The Kelvin wave initiates a boundary current that is fed from upstream by another boundary current on the left-hand wall that is established by a second Kelvin wave that propagates back upstream from the dam. The two currents are joined by a cross-channel (y) geostrophic jet at the location of the dam. For an infinitely wide channel this interior flow is just the one-dimensional geostrophic adjustment solution. One of the effects of nonlinearity in the presence of the boundary is the downstream advection of the potential vorticity front established by the fluid depths at $t = 0$ (Hermann et al., 1989; Tommason and Melville, 1992; Helfrich et al., 1999). The nose of the potential vorticity front on the right-hand wall moves at a speed that approaches the Kelvin wave speed from below as $h_1 \to 0$. When $h_1 = 0$, the downstream Kelvin wave and boundary current are replaced by a rotating gravity current with a blunt bore-like head (Stern et al., 1982; Griffiths and Hopfinger, 1983; Kubokawa and Hanawa, 1984; Helfrich and Mullarney, 2005).

The motion of the potential vorticity front was further analyzed by Stern and Helfrich (2002). They were able to eliminate the complications from the cross-channel jet and a stagnation point on the right wall by taking the initially deeper layer (depth H) to extend only a finite transverse (y) distance from right wall and upstream of $x = 0$, and taking the channel width to be infinite. Outside of this deep layer, the ambient fluid again had a depth $h_1 < H$. The long-time nonlinear evolution of the potential vorticity front intrusion was found using a long-wave, or semigeostrophic, shallow-water theory. After release of the layer, only the right-hand wall Kelvin wave and boundary current remained. The current was fed from upstream by flow parallel to the wall formed by geostrophic adjustment of the transverse step in layer depth. They also used numerical solutions of the shallow-water equations and laboratory experiments to test the theory and explore the effects of a finite-depth lower layer.

The objective is to extend the analysis in Stern and Helfrich (2002) to the case where the depth of the shallow, ambient layer is zero ($h_1 = 0$). Of interest is the development of the geostrophically adjusted flow when the initial layer has finite length along the wall. The situation to be considered is depicted in Fig. 1a. A layer of initially motionless, buoyant fluid with density ρ, depth H, width w_0 and length L is held adjacent to a vertical wall running in the x-direction. The system is rotating about the vertical axis with frequency $f/2$. The lower layer has density $\rho + \Delta\rho$ and is taken to be deep and motionless.

First consider the infinitely long case $L \to \infty$. Once released, the layer will spread offshore ($y > 0$) due to gravity until rotation begins to arrest the motion on a timescale $\sim f^{-1}$. Ignoring for the moment high-frequency inertia–gravity waves excited by the release, conservation of volume