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Original paper

A multivariate Bernstein copula model for permeability stochastic simulation

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Resumen

En este trabajo se presenta un método no paramétrico general de simulación estocástica conjunta de propiedades petrofísicas utilizando la cópula Bernstein. Este método consiste básicamente generar simulaciones estocásticas de una determinada propiedad petrofísica (variable primaria) modelando la dependencia empírica subyacente con otras propiedades petrofísicas (variables secundarias), mientras también es reproducida la dependencia espacial de la primera.
Este enfoque multivariado provee una herramienta muy flexible para modelar las complejas relaciones de dependencia de las propiedades petrofísicas. Tiene varias ventajas sobre otros métodos tradicionales, ya que no se limita al caso de la dependencia lineal entre las variables, y tampoco requiere de la suposición de normalidad y/o existencia de momentos.
En este trabajo este método es aplicado para simular un perfil de permeabilidad utilizando la porosidad vugular y velocidad de onda de corte (Ondas S) como covariables, en una formación carbonatada de doble porosidad a escala de pozo. Los valores simulados de la permeabilidad muestran un alto grado de precisión en comparación con los valores reales.

Palabras clave: permeabilidad, porosidad, velocidad de onda de corte, dependencia multivariada, cópula de Bernstein, simulación geoestadística.

Abstract

This paper introduces a general nonparametric method for joint stochastic simulation of petrophysical properties using the Bernstein copula. This method consists basically in generating stochastic simulations of a given petrophysical property (primary variable) modeling the underlying empirical dependence with other petrophysical properties (secondary variables) while reproducing the spatial dependence of the first one.
This multivariate approach provides a very flexible tool to model the complex dependence relationships of petrophysical properties. It has several advantages over other traditional methods, since it is not restricted to the case of linear dependence among variables, it does not require the assumption of normality and/or existence of moments.
In this paper this method is applied to simulate rock permeability using Vugular Porosity and Shear Wave Velocity (S-Waves) as covariates in a carbonate double-porosity formation at well log scale. Simulated permeability values show a high degree of accuracy compared to the actual values.

Key words: permeability, porosity, shear wave velocity, multivariate dependence, Bernstein copula, geostatistical simulation.

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Introduction

Integrated Reservoir Modeling (IRM) is the most accepted way to obtain the spatial distribution of petrophysical properties in oilfields (Cosentino, 2001). An important and common task performed in this method is the estimation of permeability, because it is well known that this petrophysical property is quite informative about the oil flux patterns in a reservoir. However, it is difficult to obtain direct information about permeability, and therefore it is necessary to find models of dependence with another petrophysical property (such as porosity, water saturation, etc.) in order to have an estimation of its profile (Landa *et al*., 1996).

The linear regression approach is the most common way to model permeability values using other petrophysical properties as covariates (Balan *et al*., 1995). To meet the requirement of linearity it is common to perform transformations which imply that the final result could be biased when it is back transformed; there are approaches that, in order to fit linear models, apply logarithmic transformations to induce this behavior, for example, to use the Cokriging method it is necessary to have a linear relationship since it requires the linear corregionalization model (Sanjay and Journel 1994).

The main disadvantage of linear dependency models is their lack of ability to capture and model the dependence structure or pattern (Al-Harthy *et al*. 2005). In other words, traditional methods cannot capture the complex variability of data, in terms of variance or standard deviation; hence, the predicted permeability will not reproduce extreme values of the real data. In other words, these approaches will not be able to represent impermeable barriers or high permeability zones, and from a fluid flow point of view this aspects are the most important characteristics that determine the patterns of fluid movement. In this context, the predicted permeability profile, using linear estimators, will not be an effective approximation due to its oversmoothing nature.

On the other hand, model-free function estimators like artificial neural networks are very flexible tools that have been used to model permeability. However, neural networks have some disadvantages too. First, the training process has to be done with caution and can be a lengthy process. The good results obtained by this technique are reached using a comprehensive training data set, which is not always available. On the other hand, failing in correctly calibrating the network may result in aberrant results. Another point to take into consideration is that the methodology is not yet an "off the shelf" application and requires expertise by the geoscientist (Cosentino, 2001).

Another alternative are Bayesian methods; however, the traditional framework of the Bayesian analysis is based on the multivariate normal distribution where the lower and upper tails are symmetrical. Armstrong *et al*. (2004) proposed an alternative Bayesian analysis that is based on Archimedean copulas where the joint distribution does not have to be normal and there is flexibility to have a lower tail or upper tail dependence based on the specific type of copula.

Constructing numerical models of the reservoir that honor all available data (core measurements, well logs, seismic and geological interpretations, etc.) having sparse knowledge of rock properties, leads us to consider the stochastic simulation approach (Deutsch, 1992). This is not a new concept (Haldorsen and Damsleth, 1990; Journel and Alabert, 1990), stochastic models of physical systems are used extensively in many disciplines.

Stochastic simulation is the process of building alternate, equally probable models of the spatial distribution of a random function. It is said that a simulation is conditional if the resulting realizations honor the raw data values at their locations. The most straightforward algorithms for generating realizations of a multivariate Gaussian field is provided by Sequential Gaussian Simulation (SGS) and Sequential Indicator Simulation (SIS), which are extensively used to perform permeability simulations (Holden *et al*. 1995). However despite of their improvements (Journel and Zhu, 1990; Suro-Perez, 1990) these methods are limited to cases when the spatial continuity is characterized by stationary two-point statistics and to data that is defined on the same support (Deutsch, 1992).

A competitive and more systematic method for predicting permeability may be achieved by applying stochastic joint simulations, in which the correct specification of dependence pattern between petrophysical properties is crucial (Deutsch and Cockerham, 1994). According to Deutsch this approach basically consists of an annealing geostatistical cosimulation of porosity-permeability using their empirical joint distribution.